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PB-254 314

A SIMULATION ANALYSIS OF U.S. ENERGY DEMAND, SUPPLY, AND PRICES

RAND Corporation

PREPARED FOR
National Science Foundation

October 1975

191068

NSF-RA-N75301

PB 254 314

A SIMULATION ANALYSIS OF U.S. ENERGY DEMAND, SUPPLY, AND PRICES

PREPARED UNDER GRANTS FROM THE NATIONAL SCIENCE FOUNDATION
WITH SUPPORT FROM THE ENVIRONMENTAL PROTECTION AGENCY

KENT P. ANDERSON R-1591-NSF/EPA OCTOBER 1975

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This report was prepared with the support of the National Science Foundation, under grants GI-44 and SIA-74-18660, and with the additional support of the Environmental Protection Agency. Any opinions, findings, conclusions, or recommendations expressed herein are those of the authors and do not necessarily reflect the views of NSF or EPA.

Published by The Rand Corporation

Cover Photograph from Hale Observatories
Solar corona photographed at the total eclipse
of June 8, 1918, Green River, Wyoming

BIBLIOGRAPHIC DATA SHEET	1. Report No. NSF-RA-N-75-301	2.	3. Recipient's Accession No. PB254314
4. Title and Subtitle A Simulation Analysis of U.S.Energy Demand, Supply, and Prices			5. Report Date October 1975 6.
7. Author(s) K.P. Anderson			8. Performing Organization Rept. No. R-1591-NSF/EPA
9. Performing Organization Name and Address Rand Corporation Santa Monica, CA 90406			10. Project/Task/Work Unit No. 11. Contract/Grant No. SIA-74-18660 GI 44
12. Sponsoring Organization Name and Address Research Applied to National Needs (RANN) National Science Foundation Washington, D.C. 20550			13. Type of Report & Period Covered 14.

15. Supplementary Notes

Also supported by a grant from the Environmental Protection Agency.

16. Abstracts This report describes the development of a simulation model of U.S. energy demand, supply and price, and comments on the results obtained in initial test runs. Details of the model are included for use by researchers familiar with common current specifications of supply and demand functions. The model includes the following user demand sectors and energy types: (1) residential (electricity, gas, oil); (2) commercial (electricity, gas, oil); (3) industrial (electricity, gas, distillate and residual oils); (4) transportation (petroleum products); and (5) residual (coal, electricity, gas, oil). The model contains the following supply activities: (1) coal; (2) petroleum; (3) gas; and (4) electricity. In addition several energy supply and demand projections are provided for analysts interested in the application of models to energy problems.

17. Key Words and Document Analysis. 17a. Descriptors

Demand (Economics)
Supply (Economics)
Prices
Energy
Models
Natural Gas
Coal
Oil
Numerical Analysis

17b. Identifiers/Open-Ended Terms

Energy Demand
Policy Analysis

17c. COSATI Field Group

PRICES SUBJECT TO CHANGE

18. Availability Statement
NTIS

19. Security Class (This Report) UNCLASSIFIED — 21. No. of Pages 106

20. Security Class (This Page) UNCLASSIFIED — 22. Price 5.50 2.25

FORM NTIS-35 (REV. 10-73) ENDORSED BY ANSI AND UNESCO. THIS FORM MAY BE REPRODUCED USCOMM-DC 9265-P74

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Rand
SANTA MONICA, CA. 90406

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PREFACE

This report describes the development of a simulation model of U.S. energy demand, supply, and price, and comments on the results obtained in initial test runs. Work on the model was supported by a grant from the National Science Foundation, as part of Rand's continuing research program on energy demand and conservation, and was also funded by the Environmental Protection Agency.

Although further work remains to be done before the model can be considered a routine tool for prediction and policy analysis, documentation of its present status seems appropriate to stimulate discussion of the model, in particular, and of methods of energy modeling, in general. Details of the model are included for use by researchers familiar with common current specifications of supply and demand functions. In addition, several energy supply and demand projections are provided for analysts interested in the application of models to energy problems. Much of the supply information contained here is based on that in a companion volume *The Long-Run Marginal Costs of Energy*, R-1590-NSF, by Kent P. Anderson and James C. DeHaven.

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SUMMARY

The model described in this report has several major characteristics:

- Short-run and long-run demand and supply mechanisms are treated explicitly in a dynamic model because of the importance of lagged long-run responses to changed conditions.
- Supply relationships rely heavily on technical data concerning costs and resource endowments owing to the limitations of econometric models in this area.
- The model is viewed as an analytical device for translating assumptions about market structure into market outcomes. It is capable of accommodating a wide variety of assumptions about parameter values, future technologies, projected economic and demographic growth, and foreign oil supply conditions.

The model is partly recursive and partly simultaneous. Annual levels of energy production, imports, consumption, and prices are determined simultaneously in an iterative routine. The outcome for any particular year determines long-run marginal costs (LRMCs) and values for various parameters that are held constant during the following year. All prices and costs are expressed in terms of constant 1972 dollars. Demand and supply elasticities, economic and population growth, and maximum feasible rates of expansion along the LRMC curves are given exogenously. For gas and oil, either import coefficients or import prices must be specified outside the model. The basic version specifies coefficients for liquified natural gas (LNG) and pipeline gas imports, and prices for crude and refined oil imports. The focus of the model is national. The model is currently operational on the Rand IBM 370/158. The current version requires 120K of storage, and computer costs for a typical run have been averaging less than $10. Programming costs are, of course, much more expensive.

Several modifications of the present model are under consideration: (1) updating the base period of the model and changing certain structural parameters and specifications; (2) estimating improved energy-demand elasticities for the industrial sector and incorporating into the model the relationships among energy inputs, capital and labor inputs, and industrial output; and (3) expanding the model to include several of the major measures of national economic activity, such as gross national product and personal income, that must now be specified exogenously.

DEMAND

The model includes the following user sectors and energy types:

- Residential--electricity, gas, oil.
- Commercial--electricity, gas, oil.
- Industrial--electricity, gas, distillate and residual oils.
- Transportation--petroleum products only, but further broken down to motor vehicles, civilian air transportation, railroads, pipelines, waterways.
- Residual--coal, electricity, gas, oil.

Most of the demand equations are based on econometric studies; in some cases, however, the estimated coefficients have been modified. The only major exception to the use of econometric results occurs in the commercial sector, because the pertinent historical data are inadequate for econometric analyses. In this sector, an engineering simulation approach is used.

SUPPLY

The model contains the following supply activities:

- Coal--eastern and western United States.
- Petroleum--continental United States and Alaska, net imports (crude and refined), syncrude (from coal and shale.

- Gas--continental United States and Alaska, net imports (LNG and pipeline), syngas from coal.
- Electricity--nuclear, coal, oil, gas, other.

Owing to the significant changes in technology, environmental constraints, and resource availability probable in the next two decades, the model relies very little on econometric evidence to define supply relationships. As of 1975, econometric analyses have proved incapable of dealing realistically with the kinds of structural change noted above. Instead, the supply equations are designed to use technical industrial data on projected LRMCs of production as conditioned by the level of technology, environmental protection requirements, industry organization and taxation, and resource depletion.[1] Econometric analysis and historical data are relied on for determination of short-run supply elasticities and permissible rates of expansion along the LRMC curves.

RESULTS

All of the results reported here assume the same demand parameters. The principal exogenous factors affecting consumption are shown below, along with their projected annual percentage growth rates, which are held constant throughout the simulation period, 1973 to 1995:

Exogenous Factors	*Growth Rates*
Gross national product	3.88
Value added in the industrial sector	3.50
Commercial-sector floor space	4.50-5.20
Personal income per household	2.24
Personal income per capita	2.91
Number of households	1.60
Population	0.94

The results fall into two major groups, according to the conditions assumed for the supply of gas, oil, coal, and electricity--a

[1] Most of the supply information is documented in Kent P. Anderson and James C. DeHaven, *The Long-Run Marginal Costs of Energy*, The Rand Corporation, R-1590-NSF, February 1975.

base-case group and a pessimistic-case group. The parameter values chosen for the latter impose lower short-run elasticities of supply, higher (and in some sectors more steeply sloped) LRMC curves, and lower levels for total gas and oil resource availability than in the base case. The maximum feasible growth rates for electricity as a whole, for nuclear power, and for continental U.S. gas production are cut back, and the rate of LNG imports is halved. To preserve the viability of market equilibrium, the maximum feasible growth rates for eastern and western coal are given larger values in the early years than they are in the base case. Although coal production is permitted to expand more freely, however, it must still move along higher LRMC curves.

Within each group of cases, two types of variations are tried. The first type varies the delivered prices of imported crude. Variations of 5, 9, and 11 dollars per barrel are used for both the pessimistic and the base cases. The second type is a change in the pricing of utility gas and electricity. The "rate reform" variations assign a zero weight to historical costs, in contrast to the others, which give gradually decreasing emphasis to historical costs.

AGGREGATE ENERGY

Table S-1 summarizes the results for aggregate energy across the seven variations. Total energy is measured according to the conventions used by the Bureau of Mines for calculating "gross consumption of energy resources." The total energy bill is the total cost of energy--including transportation, distribution, and taxes--to energy users. Energy imports (measured net of exports) include crude oil, refined petroleum products, pipeline gas, and LNG. (The United States is assumed to remain a net exporter of coal and electricity. These exports are included in the residual demand categories.) The total cost of imports is the delivered cost, p.o.e.

The figures in the table show that between 1973 and 1985 total energy use increases by a factor ranging from 32 to 46 percent, corresponding to average annual growth rates of 2.3 to 3.2 percent. Since the model incorporates a real GNP growth rate of nearly 3.9 percent,

Table S-1

ENERGY CONSUMPTION AND IMPORTS, SELECTED YEARS--1973-1995

Variable	Initial Value						
Base year, 1973							
Total energy	75.6 (10^{15} Btu)						
Cost of energy	110.4 (10^{9} $)						
Unit cost of energy	1.46 ($/$10^{6}$ Btu)						
Energy imports	12.5 (10^{15} Btu)						
Cost of imports	8.8 (10^{9} $)						
	Projected Value						
	Base Case[a]				Pessimistic Case[a]		
	$5	$9	$11	$9/RR	$5	$11	$9/RR
1980							
Total energy	96.1	93.9	93.0	89.5	93.3	91.2	87.9
Cost of energy	163.3	184.8	194.2	192.2	184.3	197.3	157.4
Unit cost of energy	1.70	1.97	2.09	2.15	1.98	2.16	2.25
Energy imports	13.3	6.5	4.7	7.6	10.0	6.4	9.5
Cost of imports	13.9	11.7	9.9	13.2	15.1	13.0	15.7
1985							
Total energy	110.4	109.9	109.3	103.6	106.3	104.8	99.6
Cost of energy	204.2	219.6	228.1	229.5	225.0	232.6	239.4
Unit cost of energy	1.85	2.00	2.09	2.22	2.12	2.22	2.40
Energy imports	17.6	6.5	3.8	8.6	10.7	6.5	12.4
Cost of imports	21.0	12.4	8.6	15.5	18.0	13.3	20.6
1995							
Total energy	140.5	139.2	137.9	133.4	134.0	133.5	127.5
Cost of energy	320.8	330.0	342.7	339.0	350.8	352.3	362.1
Unit cost of energy	2.28	2.37	2.49	2.54	2.62	2.64	2.84
Energy imports	26.3	15.1	10.1	18.1	19.7	16.3	23.4
Cost of imports	39.2	27.5	22.0	31.8	38.6	33.1	45.5

[a]Prices are for one barrel of crude. "RR" stands for "rate reform."

these values imply an annual rate of decline of 0.7 to 1.5 percent in the energy/GNP ratio. This contrasts with a nearly zero trend rate of decline, as indicated by weighted averages for the years 1951 to 1953 and 1971 to 1973, and reflects both higher-than-historic energy prices and low estimated elasticities of demand with respect to income or output in some sectors.

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I. INTRODUCTION

Public apathy about the adequacy of energy supply was not swept away by the "Great East Coast Blackout" of October 1965, but it began to melt noticeably thereafter, accelerated by the continuing summertime difficulties of such electric utilities as New York City's Consolidated Edison Company. The *coup de grâce* was of course delivered by the Organization of Petroleum Exporting Countries some eight years later. Temporary shortages of gasoline and fuel oil followed by sharply increased fuel and electricity prices demonstrated in concrete fashion that energy supplies may be no less evanescent than California gold and in the end far more valuable.

A gradual increase in concern about environmental quality has complicated the issue of energy supply adequacy, since energy production and use inevitably contribute to air, water, soil, and aesthetic pollution. Energy-industry executives fear that environmental restrictions will unjustifiably reduce profits and prevent their firms from meeting future demands at reasonable prices. Environmentalists fear that continued exploitation of energy resources will result in gross environmental insults and irreparable social losses.

The consuming public and its governmental representatives must now decide how to balance the desire for abundant energy against the goal of improved environmental quality as well as competing claims on the nation's resources. The model and simulation analysis of energy demand and supply presented in this report are intended to guide those who are now attempting to determine the appropriate actions for government. Section II describes the model; Sec. III summarizes the results and conclusions of the initial simulation analyses.

II. THE MODEL

The model is a dynamic representation of the interactions among the supply sectors and the demand sectors of the U.S. energy market. Given exogenous estimates of the macroeconomic conditions in the country and a supply schedule for international oil, the model simulates the determination of market clearing prices and quantities for all forms of energy. It is an annual model with stock and flow adjustments in both the demand and supply sectors. Each year, demand and supply conditions are changed and markets are cleared because of activities that occurred in prior years.[1]

The supply side of the model covers coal (eastern and western United States); natural gas (continental United States and Alaska, synthetic, pipeline imports, and liquified natural gas (LNG) imports); petroleum (continental United States and Alaska, synthetic, shale, and crude and refined imports); and electricity (produced from coal, gas, oil, nuclear fuel, and hydro and geothermal sources). Energy supplied from the fossil-fuel sources depends on total reserves, cumulative additions to reserves, depletion rates, current and past capacity levels, and the actual levels of current and past production. These concepts are combined to define long-run and short-run marginal cost curves for each domestic source of each fossil fuel. Production rates of electricity and nuclear fuel are based on long-run marginal costs (LRMCs), but with less detail. The supplies of synthetic fuels and imports are based on simpler assumptions.

The demand side of the model is divided into industrial, commercial, residential, and transportation sectors, and a residual. The industrial, commercial, and residential sectors consume electricity, natural gas, and petroleum products. The transportation sector is considered to consume only petroleum products. The residual demand

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[1]The computer program for the model is written in FORTRAN for operation on Rand's IBM 370/158 system. It requires 120K of storage, and a complete run--simulating the operation of energy markets each year from 1973 to 1995--typically requires less than $10 of computer time.

covers all unallocated uses of the above fuels and all coal demand except the use of coal by electric utilities. All the demand sectors are based on econometrically estimated fuel coefficients and elasticities except for the commercial sector. It is represented by a simulation model of energy use derived from Rand's work on energy conservation in the commercial sector.

The purpose of the model is to trace the dynamic interactions among the different energy sources and uses and to estimate the time path of energy prices that would equate supply and demand.[1] The model results from a design philosophy that includes several prescriptions:

- Short-run and long-run demand and supply mechanisms should be treated explicitly in a dynamic model because of the importance of lagged long-run responses to changed conditions.
- Supply relationships should rely heavily on technical data concerning costs and resource endowments owing to the limitations of econometric models in this area.
- The model should be viewed as an analytical device for translating assumptions about market structure into market outcomes. It should, therefore, be capable of accommodating a wide variety of assumptions about parameter values, future technologies, projected economic and demographic growth, and foreign oil supply conditions. Only in this way can it be expected to deal effectively with the uncertainties of the future.

The structure of the model reflects these prescriptions. It is partly recursive and partly simultaneous. Annual levels of energy production, imports, consumption, and prices are determined simultaneously

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[1]The relationship of this model to other energy models is discussed in Appendix C. The Rand model is compared with the Federal Energy Administration model, the Brookhaven model, and the Hudson-Jorgenson model.

in an iterative routine. The outcome for any particular year determines LRMCs and values for various parameters that are held constant during the following year. All prices and costs are expressed in terms of constant 1972 dollars. Demand and supply elasticities, economic and population growth, and maximum feasible rates of expansion along the LRMC curves are given exogenously.[1] For gas and oil, either import coefficients or import prices must be specified outside the model. The basic version specifies coefficients for liquified natural gas (LNC) and pipeline gas imports, and prices for crude and refined oil imports. The focus of the model is national, with only a few regional distinctions. Appendix A gives a full specification. The salient features are described below.

DEMAND

The model includes the following user sectors and energy types:

- Residential--electricity, gas, oil.
- Commercial--electricity, gas, oil.
- Industrial--electricity, gas, distillate and residual oils.
- Transportation--petroleum products only, but further broken down to motor vehicles, civilian air transportation, railroads, pipelines, waterways.
- Residual--coal, electricity, gas, oil.

Most of the demand equations are based on econometric studies; in some cases, however, the estimated coefficients have been modified.[2] The

---

[1]Several modifications of the present model are under consideration: (1) updating the base period of the model and changing certain structural parameters and specifications; (2) estimating improved energy-demand elasticities for the industrial sector and incorporating into the model the relationships among energy inputs, capital and labor inputs; and industrial output; and (3) expanding the model to include several of the major measures of national economic activity, such as gross national product and personal income, that must now be specified exogenously.

[2]The coefficients I have used appear in Appendix B. The econometric studies on which I have relied in making my judgments are cited throughout the text.

only major exception to the use of econometric results occurs in the commercial sector, because the pertinent historical data are inadequate for econometric analyses. In this sector, an engineering simulation approach is used.

In the residential sector, the demand equations are of the following form:

$$D_t = (D_{t-1})^{\eta_d} (D_t^*)^{1-\eta_d} \qquad (1)$$

and

$$D_t^* = \eta_o \cdot \prod_{i=1}^{m} P_{it}^{\eta_{pi}} \cdot y_t^{\eta_y} \cdot h_t^{\eta_h} \cdot n_t \,, \qquad (2)$$

where D = actual demand,

t = year,

D^* = target demand,

m = number of energy types,

P_i = price of energy type i,

y = income per household,

h = persons per household,

n = number of households,

η = constant.

Equations (1) and (2) constitute a familiar type of lagged-adjustment model. If η_d is zero, adjustment to target demand is immediate. The closer η_d is to unity, the longer is the time required to achieve adjustment to a given percentage of the target level. The η_i's are long-run price elasticities of demand, and η_y is the long-run income elasticity of demand. Short-run elasticities are $(1 - \eta_d)$ times the long-run elasticities. Estimates for the η's are based primarily on results reported in Ref. 1. The value for the lag coefficient η_d (for electricity) is taken from the work of Mount, Chapman, and Tyrrell [2].[1] Similar values are assumed to apply to gas and oil. The income and own-price coefficients for oil demand are based partly on results obtained by Erikson, et al. [3].

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[1] The lag coefficient was estimated by ordinary least squares in a constant elasticity model similar to that in the text.

The commercial-sector demand model is of the following form for each energy type:

$$D_t = \eta_d D_{t-1} + \left(\frac{P_t}{P_o}\right)^{\eta_p} \cdot ND_t^* \qquad (3)$$

and

$$ND_t^* = \sum_{i=1}^{5} \left(A_t^i - \eta_d A_{t-1}^i\right) \cdot \left(\sum_{j=1}^{3} b_{jt}^i \cdot s_j^i t\right), \qquad (4)$$

where the variables heretofore undefined are as shown below:

ND^* = potential "new" or "free" demand,
A^i = commercial floor space in region i,
b_j^i = input coefficient for a given type of energy in region i using system configuration j,
s_j^i = share of new plus renovated commercial floor space in region i using system configuration j.

As in the case of the residential sector, Eqs. (3) and (4) form a lagged-adjustment model of demand. The coefficient, η_p, is the long-run usage-level elasticity of demand with respect to own-price. The short-run elasticity is $\left[1 - \eta_d(D_{t-1}/D_t)\right] \cdot \eta_p$. Estimates for the η_p's are guesses based in part on econometric studies of the residential sector and of the commercial and residential sectors combined.[1] The lag coefficient, η_d, is a best guess of the current rate of building replacement and renovation in the commercial sector, based on historical construction statistics. The b_j^i's and s_j^i's are determined in a separate subroutine for each of three system configurations--all-electric, conventional (electric air conditioning with fossil-fuel heating), and total energy (on-site electricity generation with waste-heate recovery)--in five regions of the country.[2] The b_j^i's are fixed numbers, except

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[1] See Refs. 1 and 3.

[2] These regions are combinations of the Bureau-of-the Census regions, as follows: (1) New England, Middle Atlantic; (2) East North Central, West North Central; (3) South Atlantic, East South Central, West South Central; (4) Mountain; (5) Pacific.

for a price-based allocation of fossil-fuel usage between gas and oil. The s_j^i's are determined by the relative system costs of each of the three energy-use configurations in each region with 80 percent of new or renovated floor space going to the cheapest system and 10 percent to each of the other two. These small shares assigned to the more costly systems are assumed to reflect the combined effects of error and variations in operating conditions or costs within regions. Both the trade-off curves used to determine the s_j^i's and the energy-use coefficients are based on engineering cost analyses of representative commercial buildings.[1]

The demand model for the industrial sector includes the following equations for each of two energy classes, gas and oil together and electricity:[2]

$$D_t = (D_{t-1})^{\eta_d} \cdot (D_t^*)^{1-\eta_d} \tag{5}$$

and

$$D_t^* = \eta_o \prod_{i=1}^{m} P_{it}^{\eta_{pi}} \cdot V_t^{\eta_v} , \tag{6}$$

where V = the real value added in the industrial sector. Equations (5) and (6) constitute a system analogous to that for the residential sector, and the remarks about adjustment lags and elasticities made above apply to the industrial sector as well. Estimates for the coefficients of Eq. (6) are based on the results reported in Fig. V-2 of Ref. 3. The adjustment coefficient in Eq. (5) has a best-guess value of 0.9.[3] The fuel-split model for gas and oil is of the form

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[1] See Ref. 4 for a description of this work.

[2] The initial specification of the model contained separate equations for oil and gas, but the estimated coefficients implied an unacceptable pattern of own- and cross-price effects. This pattern was made more plausible by (1) aggregating the elasticity coefficients for oil and gas using as weights the relative shares of oil and gas consumed in 1972, and then (2) introducing a fuel-split model for gas and oil.

[3] Mount, Chapman, and Tyrrell [2] obtain a value of 0.89 for industrial electricity demand using a similar model.

$$F_t = (F_{t-1})^{\eta_f} \cdot (F_t^*)^{1-\eta_f} , \tag{7}$$

$$F_t^* = \frac{D_0^{gas}}{D_0^{oil}} \cdot \left(\frac{p_t^{gas}/p_0^{gas}}{p_t^{oil}/p_0^{oil}} \right)^{-\eta_{g/o}} , \tag{8}$$

$$D_t^{oil} = \frac{D_t^{gas + oil}}{1 + F_t} , \tag{9}$$

$$D_t^{gas} = D_t^{gas + oil} - D_t^{oil} , \tag{10}$$

where F is the actual ratio of gas-to-oil demand and F^* is the target ratio. Best-guess values for η_f and $\eta_{g/o}$ are 0.9 and 3.0, respectively--the value of η_f matching that for η_d, and that for $\eta_{g/o}$ loosely reflecting fuel-split results reported for commercial and household demand in Ref. 5.

The demands of the transportation sector are explicitly represented for petroleum products only. Direct consumption of other types of energy is small and is lumped into the residual categories for such energy. The transportation sector is broken down further into several subsections, the most important of which is motor vehicle demand. The following three equations adopted from the work of Burright and Enns reported in Ref. 6 constitute the demand model for the motor vehicle sector:[1]

$$M_t = \left[\eta_0 + \eta_y \cdot y + \eta_g \cdot p_{g,t-1} \right] \cdot n_t + \eta_d M_{t-1} , \tag{11}$$

$$MPG_t = \left(p_{g,t-1}/p_{go} \right)^{\eta_p} , \tag{12}$$

$$D_t = \eta_0 \cdot p_{gt}^{\eta_g} \cdot \left(y_t/h_t \right)^{\eta_y} \cdot \left(\frac{M_t}{P_t} \right)^{\eta_m} \cdot MPG_t^{\eta_{mpg}} \cdot P_t , \tag{13}$$

---

[1] Equations (11) and (12) are derived from Burright's work. Equation (13) is taken from Enn's results. Since Burright's equations pertain only to the stock of automobiles, whereas Enn's apply to the total

where all previously introduced variables are defined as above and

M = stock of motor vehicles,

p_g = price of regular gasoline,

MPG = average miles per gallon of motor vehicle fuel,

P = population.

As in the sectors described above, demand is represented by a lagged-adjustment model. In this case, the rate of turnover of the motor vehicle stock is the effective constraint on short-run responsiveness. The lag coefficient, η_d, reflects both the rate of vehicle scrappage and the influence of the amount of the existing stock on new-vehicle purchases.[1]

The demand of the remaining transportation-sector components are generally of the form

$$D_t = \eta_o \cdot \prod_{i=1}^{m} p_{it}^{\eta_i} \cdot (y_t/h_t)^{\eta_y} \cdot p_t \cdot \qquad (14)$$

The price-elasticity coefficients do not reflect usage-intensity adjustments for the transportation modes, since no suitable estimates for such responses could be found. Rather they are derived from the price elasticities of demand for the transportation services themselves, using as weights the share of average total cost due to fuel. The equations have no explicit lag structure. The regression analyses of national time-series data on which they are based contain no explicit dynamic, and it is not clear whether they measure purely short-run or

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stock of motor vehicles, it is necessary to assume that the size and average gasoline mileage of the two stocks are proportional. Automobiles have accounted for a nearly constant share of slightly more than 80 percent of the motor vehicle stock since the 1950s. The assumption of stock proportionality therefore seems reasonable. Whether the assumption of mileage proportionality is a satisfactory simplification is less clear. See Wildhorn, et al. [6].

[1] I assume the scrappage rate to be 0.1, which is the rate that would be observed if the vehicle stock were composed of vehicles lasting exactly ten years (about the average life span of an automobile in this country), and were distributed in equal amounts across age groups. The inhibiting effect of the existing stock on new-vehicle purchases is negative and thus implies a value for η_d of less than 0.9. (See Eq. Z(143) in Appendix A.)

long-run effects. The results of Lave [7] are used to develop the passenger components of rail and civilian air transport demand. Freight demands, estimated by James Stucker, are presented in Appendix D.

Residual demands for natural gas, petroleum products, and electricity are assumed to equal a given constant fraction of the other demands in their category. The residual demand for coal includes industrial, commercial, and residential use as well as net exports and is expressed as follows:

$$D_t = \eta_o \cdot GNP_t \cdot p_{ct}^{\eta_c} \cdot p_{alt,t}^{\eta_{alt}} \cdot e^{-\eta_t \cdot t}, \qquad (15)$$

where GNP = real gross national product,

p_c = price of coal to industrial users,

p_{alt} = weighted average of prices of gas and oil to industrial users.

As in the case of nonvehicular transportation-energy use, no explicit lag structure is contained in Eq. (15). A simple time-series regression of annual national data forms the basis for the residual coal equation, and the coefficients may reflect a mixture of short-run and long-run effects.[1]

SUPPLY

The model contains the following supply activities:

- Coal--eastern and western United States.
- Petroleum--continental United States and Alaska, net imports (crude and refined), syncrude (from coal and shale).
- Gas--continental United States and Alaska, net imports (LNG and pipeline), syngas from coal.
- Electricity--nuclear, coal, oil, gas, other.

[1] See Appendix E for details on the estimation of Eq. (15).

Owing to the significant changes in technology, environmental constraints, and resource availability probable in the next two decades, the model relies very little on econometric evidence to define supply relationships. As of 1975, econometric analyses have proved incapable of dealing realistically with the kinds of structural change noted above. Instead, the supply equations are designed to use technical industrial data on the LRMCs of production as conditioned by the level of technology, environmental protection requirements, industry organization and taxation, and resource depletion.[1] Econometric analysis and historical data are relied on for determination of short-run supply elasticities and permissible rates of expansion along the LRMC curves.

Figure 1 shows the domestic supply mechanism for depletable energy resources--e.g., coal, natural gas, and crude petroleum. Q is actual output; $\bar{Q}$ is *ex-ante* full-capacity output. R is cumulative production to January 1 of the year in question; k is LRMC; and c is short-run marginal cost (SRMC). Cumulative production or additions to reserves (in the case of gas and oil) determine the level of k, which in turn determines the height of the SRMC curve. At $\bar{Q}$, k = c. The hatched line shows how costs affect pricing in the model. If $Q < \bar{Q}$, the LRMC of continental U.S. production sets a floor on price. Output is reduced to a level sufficient to maintain that price.[2] If $Q > \bar{Q}$, price is set equal to SRMC. The SRMCs of all alternative sources of supply

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[1] See Ref. 8, a companion report that presents detailed information on the LRMCs of energy production.

[2] It is unlikely that energy-resource industries would allow prices to fall below the LRMC of domestic production in the absence of regulation. As indicated below in the text, the model assumes that "new" gas prices are unregulated. Oil, uranium, and coal production are also assumed to be unregulated. In the absence of regulation, prices could still fall below LRMC. In the perfectly competitive model of industry behavior, this would occur whenever demand was insufficient to absorb the full-capacity output of the industry. In an oligopolistic industry, such as the oil industry, it might not occur under any circumstances. In the case at hand, I do not believe that real-world prices are sufficiently downward flexible to permit prices below LRMC in the absence of regulation. For institutional reasons (related to future price expectations and long-run versus short-run profit maximization), I would expect most firms to cut output rather than prices in times of "insufficient" demand.

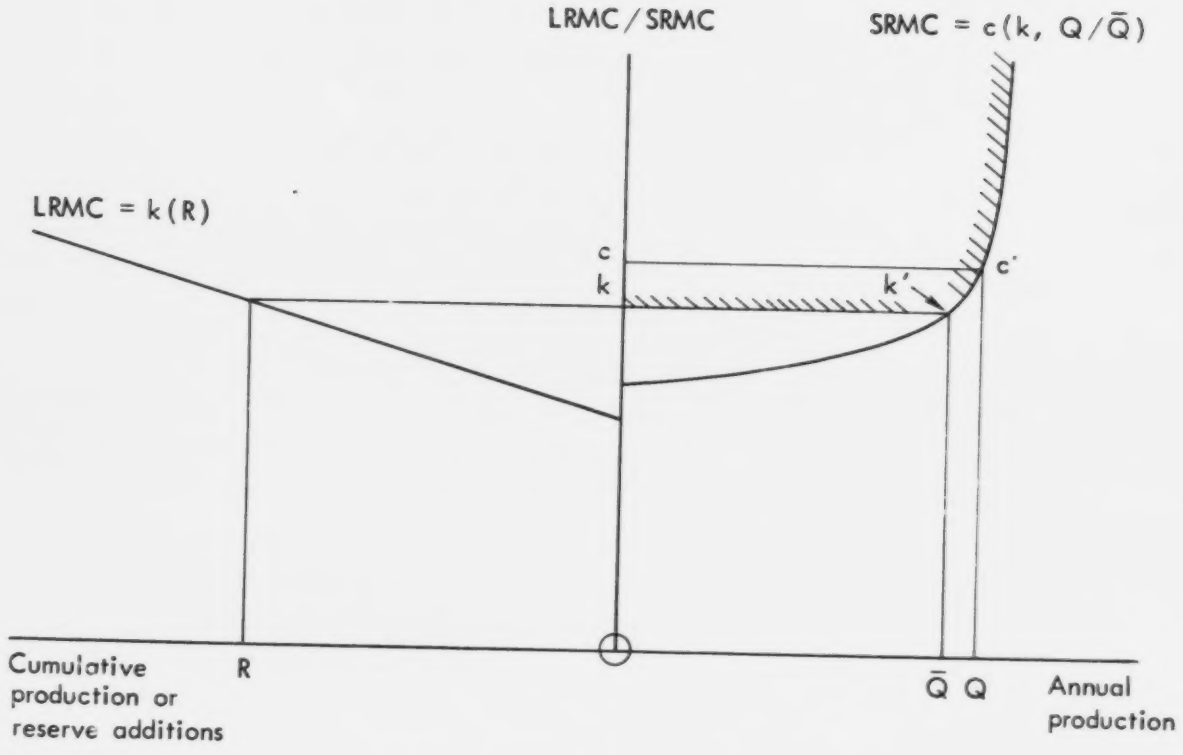

Fig. 1 — Domestic production of depletable energy resources

for each type of energy are equated, after transportation costs have been determined. Costs are therefore minimized in each supply sector, subject to the constraining floor given by LRMC. The area indicated by the points cc'k'k represents quasi-rents or temporary excess profits accruing to the producing industry. If $\bar{Q}$ has been kept unduly low by barriers to entry, a portion of this area represents monopoly or oligopoly rents.

The maximum feasible rate of expansion along the LRMC curve is expressed in terms of the growth of *ex-post* production capacity, $\tilde{Q}$, which is the greater of

$$\bar{Q} + \rho(Q-\bar{Q}) \quad , \qquad 1 \geq \rho \geq 0$$

and

$$(1 - \delta)\bar{Q} \quad , \qquad 1 \geq \delta \geq 0 \quad ,$$

where ρ is the share of production above or below $\bar{Q}$ that reflects accelerated additions to or retirements from capacity and δ is the decline rate or depreciation rate. Next year's *ex-ante* productive capacity is given by

$$(1 + r)\tilde{Q} \quad ,$$

where r is the maximum feasible rate of growth of capacity. The values chosen for r reflect such factors as historical industry growth trends, the level of cumulative additions to reserves or cumulative production relative to total resource endowment, the expected schedule of initial commercial development (in the case of synthetic fuels), input supply conditions in the industry, regulatory policy, and uncertainty.

Coal

In the coal industry, LRMC is a function of cumulative production (rather than resource additions), and r is not systematically related to resource endowments, since (1) the reserve/production ratio does not play a crucial role in determining production rates as in the case of natural gas and oil, and (2) total resource endowments are great

relative to potential production over the period considered. The LRMC curves for eastern and western coal are of the form

$$k_t = k_o + \beta\left[\sum_{i=0}^{t-1} Q_i/Q^*\right]^{\eta} , \qquad (16)$$

where Q^* equals total resources available at a LRMC of not more than $k_o + \beta$. The magnitude of η determines how rapidly this upper bound is reached. For eastern coal, η is taken to equal unity; a simple linear relationship seems proper in the absence of suitable *a priori* evidence. For western coal, η is taken to equal 3.0. This assumption has the effect of concentrating the projected increases in LRMC in the time periods where cumulative production is approaching Q^*, and appears to be reasonable in view of the large deposits of low-cost coal that await exploitation in the western United States.

Oil and Gas

The maximum feasible rate of growth of productive capacity on the LRMC curve in the gas- and oil-producing sectors is related to the level of cumulative additions to reserves, R. The latter are, in turn, determined by *ex-post* productive capacity, $\tilde{Q}$, according to the following formula:

$$R_t = R_{t-1} + \nu\left[(\tilde{Q}_t - \tilde{Q}_{t-1}) + \delta\tilde{Q}_t\right] , \qquad (17)$$

which states that cumulative additions to reserves in year t are equal to last year's value plus those required (1) to augment capacity and (2) to replace capacity lost through the natural decline of well productivity over time. The parameters ν and δ are, respectively, the marginal reserve/production ratio and the decline ratio, and satisfy the constraint

$$\nu = (1 - e^{-\delta T})/\delta , \qquad (18)$$

where T is the mean life of a well.[1]

Figure 2 shows how the permissible growth rate of capacity is related to R/R^*, where R^* is an upper bound to cumulative reserve additions determined by the state of technology and geological knowledge.[2] For values of R/R^* less than ϕ_1, r is set equal to a fixed geometric growth rate, π_1, which may vary exogenously from year to year (although for clarity it is shown as constant in the figure). If R/R^* is in the interval (ϕ_1, ϕ_2), r declines gradually to zero, the values of π_2 and δ being chosen so as to satisfy

$$\pi_1 = \pi_2 - \phi_1 \cdot \delta$$

and

$$0 = \pi_2 - \phi_2 \cdot \delta .$$

If R/R^* exceeds ϕ_2, then r is set equal to a function whose value varies between zero at $R/R^* = \phi_2$ and $-\delta$ at $R/R^* = 1$.[3]

Figure 3 shows $\tilde{Q}$ as a function of R/R^* on the simplifying assumption that $Q = \bar{Q} = \tilde{Q}$. Since R/R^* approaches unity as a limit, $\tilde{Q}$ approaches zero as a limit. Values for ϕ_1 and ϕ_2 must be determined largely by intuition, since little empirical evidence is available for guidance. In all versions of the model, ϕ_1 is 1/2 for all sources of crude oil and natural gas except Alaskan gas,[4] and ϕ_2 is 2/3.

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[1] See Adelman [9] for a description of the theoretical model from which this relationship is derived. In principle, the mean life of a well should be a function of the price of oil. Lacking sufficient data, I hold it constant.

[2] Material surveyed in Ref. 8 provides the principal basis for estimating R^*.

[3] This function satisfies the following conditions: $dR/dt = \nu(d\tilde{Q}/dt + \delta\tilde{Q}) = \alpha(R^* - R)$. The parameter α is equal to $\nu\delta Q^{max}/R^*(1-\phi_2)$, the value required to insure that dR/dt is correctly scaled at $\tilde{Q} = Q^{max}$.

[4] Owing to the relatively high values assigned to π_1 for Alaskan gas when $R/R^* < \phi_1$, the decline in the growth rate should begin at a lower level of depletion than in the other cases. Accordingly, ϕ_1 is 1/3 in this case.

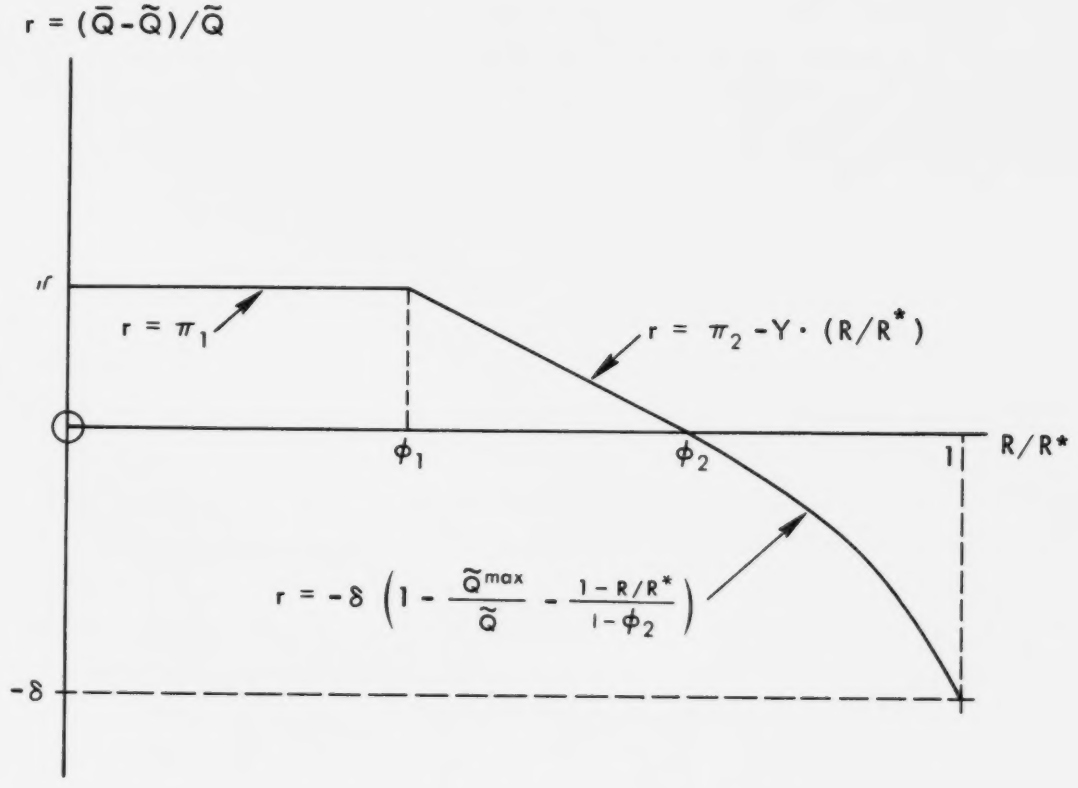

Fig. 2—Maximum feasible growth rate along the LRMC curve

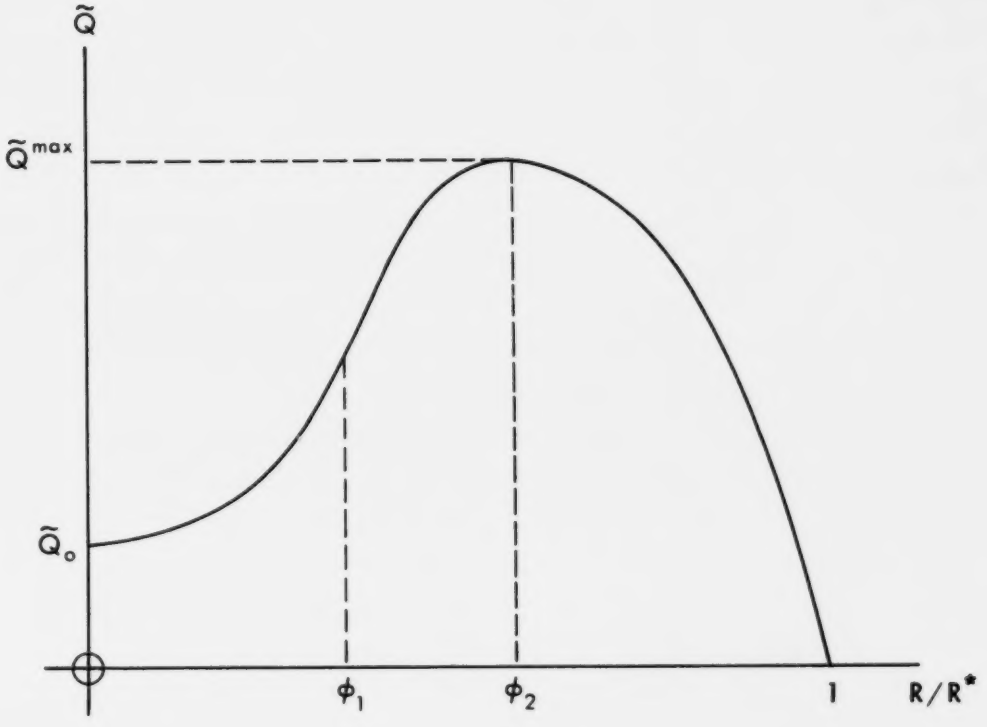

Fig. 3 — Productive capacity versus resource depletion

The LRMC curves for crude oil and natural gas are linear functions of R. As is the case for all the LRMC curves used in the model, the principal source of data is Ref. 8

Electricity

The production of electricity has no cumulative effect on its LRMC except insofar as it contributes to the depletion of the fuels burned in generation. Moreover, the SRMC curve of electricity supply has a virtually zero elasticity with respect to cost, since output is strictly limited by available generating capacity and additions to capacity are difficult to accelerate, owing to the long lead times involved. As a result of these characteristics, the relationships diagrammed in Fig. 1 are inapplicable to the case of electricity.

If electricity rates are such as to cause excessive demand when *ex-ante* capacity is $\bar{Q}$, then a flat *ad valorem* tax is applied at the same rate to all user classes until demand is restrained to a level equal to $\bar{Q}$ minus transmission and distribution losses. Since utilities are regulated and their rates tied by various formulae to historical costs or other inflexible cost measurements, it is unrealistic to assume that rates could rise to choke off demand. A tax may be a somewhat unlikely alternative, but it has the advantage of permitting the market to achieve short-run equilibrium when capacity is inadequate at prevailing rates. If demand is sufficient to exhaust *ex-ante* capacity at prevailing rates, then *ex-post* capacity is raised from last year's value just enough to meet demand.

The calculation of the LRMC of electric service for each user class takes place in several steps. First, a weighted average of the generating costs at new nuclear and coal-fired plants is used to compute a LRMC for electricity generation. In this calculation, uranium is assumed to be priced at its LRMC and coal at the current average nine-month price for the nation, or at 150 percent of the average LRMC for coal, eastern and western, whichever is less. Plant investment, maintenance, and nonfuel operating costs are increased somewhat over time, continuing the recent upward trend in the real cost of electricity production. Second, the LRMC for generation is adjusted upward to

account for the transmission and distribution (T&D) losses applicable to each user class. Third, a T&D component for each class is added to the adjusted cost of generation.

The allocation of total electricity generation to each of the principal energy-source categories--nuclear, coal, oil, gas, and other--is specified exogenously as follows:

$$\bar{Q}_t = \max \begin{cases} (1 + \pi)Q_{t-1} \\ (1 - \delta)\bar{Q}_{t-1} \end{cases} , \tag{19}$$

$$\bar{Q}_t^i = \bar{Q}_0^i + \beta^i \cdot t , \qquad i = \text{oil, gas, other}$$

$$\bar{Q}_t^n = S_n(t) \cdot \bar{Q}_t , \tag{20}$$

$$\bar{Q}_t^c = \bar{Q}_t - \sum_{i \neq c} \bar{Q}_t^i ,$$

$$Q_t^i = \left(\frac{Q_t}{\bar{Q}_t}\right) \cdot \bar{Q}_t^i , \quad Q_t \leq \bar{Q}_t , \tag{21}$$

where the barred Q's denote potential maximum production levels for the period and the unbarred Q's represent the actual *ex-post* values;[1] the superscripts indicate type of energy source; π is the maximum feasible growth rate; δ is the depreciation rate on generation capacity; and $S_n(t)$ is an exogenously specified function of time for determining the nuclear share of generating capacity.

The above formulation projects generation by oil, gas, and "other" (hydro, geothermal, etc.) as linear functions of time, using the forecasts of Refs. 10 and 11 as a basis for judgment. Generation by nuclear fuel is a time-related share of total generation,[2] and coal is a residual.

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[1] Capacity expands just enough to meet actual demand. The model does not allow accumulation of excess production capacity.

[2] The function used to compute this share generates an s-shaped curve and is fitted to three points: actual share for 1972 and AEC-

If demand is insufficient to exhaust projected generation, then projected allocations are reduced proportionately so as to meet demand exactly.

Synfuel and Shale Oil

The production of pipeline-quality gas and syncrude from coal bears a resemblance to that of electricity in that cumulative production contributes to an increased LRMC only through its effect on coal depletion. On the other hand, crude produced from shale resembles coal production. For all three production processes, the model incorporates nonzero short-run elasticities of output with respect to cost. They are assigned rather low values and are assumed to represent the industry's ability to accelerate additions to capacity. Production is restricted to zero in the early years, regardless of prices.[1]

Nuclear Fuel

The production of nuclear fuel from uranium is assumed to occur at whatever rates are required to meet the demands of the electric utility sector for both initial inventory in new reactors and replacement of spent fuel. Prices are assumed to be equal to LRMCs at all times.

Imports

Pipeline and LNG imports are given as fractions of domestic production. Through 1980, the combined total of these imports is held to 4 percent of domestic production, the 1972 figure--pipeline gas being gradually replaced by LNG, owing to a projected loss of Canadian

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projected shares for 1985 and 2000 as reported in Ref. 12. This function is scheduled to be updated periodically as the projected shares are consistently decreased. Some of our projected runs call for the function to be parameterized.

[1]Syngas is potentially available after 1975, shale oil and syncrude after 1979. Maximum feasible growth rates and potential first-year production levels are based on the "base-case" projections for these production processes in Ref. 10.

gas sources. After 1980, pipeline gas imports are zero. LNG imports climb to 11.5 percent of domestic production by 1995. These import projections are best guesses. They assign a minor role to gas imports because of the uncertain availability of Canadian gas and the high cost of LNG imports.

Net crude and refined petroleum imports equal the gap between domestic production and consumption as determined by the values assumed for oil import prices. The ratio of refined to crude imports is held constant at its 1973 level. This ratio is important in the model only for calculating the import energy bill, since imports and domestic products, both crude and refined, are constrained to have identical delivered prices.

Foreign trade in electricity and coal sectors is subsumed in their respective residual categories. No explicit accounting is made for them.

PRICING

Various aspects of pricing energy resources and electricity in the model are discussed above. Additional details are set out in this section.

Coal

The SRMCs of eastern and western coal delivered to the customer are assumed to be equalized by market competition. F.o.b. the mine, the two SRMCs differ by an amount equal to the additional cost of transporting western coal to eastern markets plus an adjustment factor for the higher heat value of eastern coal. The average price of coal to industrial customers is a weighted average of the SRMCs of eastern and western coal--the weights being production shares adjusted for shipments from the western to the eastern region, plus an intraregional transportation cost figure that includes an adjustment for the price premia on special types of coal used by industry. The average price of coal to electric utilities is a production-share-weighted average of eastern and western SRMCs plus an average transportation cost figure that includes a quality-adjustment component. Historical and

projected transportation cost data reported in Ref. 8 provide the basis for the estimates used in the model.

Electricity

The average price of electricity for each customer class is assumed to be a weighted average of the base-year price and the LRMC for the class computed in the manner outlined in the subsection "Supply." In the base case, the weight assigned to the initial-year price declines as either production increases or time passes, so that a gradual transition to a simplified form of marginal-cost pricing takes place.[1] Such a trend may soon emerge in the electric utility industry.[2] In the""rate-reform" case, the weight assigned to base-year prices is zero.

Natural Gas and Syngas

If the market for natural gas is to achieve equilibrium, it is necessary to assume either that the price of new (or all) natural gas is decontrolled, that the Federal Power Commission (FPC) correctly predicts (and imposes) an equilibrium price, or that an equilibrium tax is levied on regulated gas sales. The first of these alternatives seems to be the most likely, and indeed the FPC appears to be moving in this direction by allowing higher prices on new contracts for interstate shipment.

The model assumes that new gas prices are unregulated. The SRMCs of continental U.S. gas at the wellhead, Alaskan gas and imported pipeline gas at the point of entry into the continental United States, and synthetic gas at the plant are equalized so as to minimize the total costs of new gas. LNG imports are assumed to be brought in at a fixed price considerably above the LRMC of continental U.S. or Alaskan gas and, in most cases, above the SRMC of domestic gas. Despite their higher cost, LNG imports may be economically desirable in order to

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[1] LRMCs as calculated in the model do not properly separate capacity, energy, and customer-related costs. Moreover, they neglect the fact that true marginal-cost pricing would reflect SRMCs, not LRMCs.

[2] Reference 13 describes a recent landmark decision.

reduce the risk of short supply by providing an alternative gas source and to meet peak-period demands when domestic sources may not be available even at much-elevated prices. The average "wellhead" price of new natural and synthetic gas is a weighted average of the SRMC for domestic production and the cost of imports. The price of gas to gas-utility customers is a weighted average of the base-year price and an incremental cost figure. The latter is the average wellhead price adjusted for gas losses plus a constant equal to the estimated LRMC of transmission, distribution, and customer service. The weighting scheme used to calculate customer prices in the base case implies a gradual shift to a simplified form of marginal-cost pricing, just as in the electric utility sector. In the rate-reform case, the weight assigned to base-year prices is zero, so that rates immediately reflect marginal costs.

Crude and Refined Petroleum

In the petroleum sector, the SRMCs of all crude sources (natural and synthetic) are equalized at the price specified for crude oil imports. Refined imports have the same price as an undifferentiated barrel of refined product at domestic refineries. Prices for refined products are the SRMC of crude plus the LRMC of refining plus a factor that measures the combined effects of T&D costs, on one hand, and product-price-differentiation on the other. Syncrude and shale oil facilities are brought into production only after 1979, and then only if SRMC is at least as great as LRMC. If, subsequently, SRMC falls below LRMC, facilities are allowed to depreciate at a constant geometric rate.[1]

[1] A similar feature attaches to synthetic gas production.

III. SIMULATION RESULTS

The results described in this section are tentative, since a strictly limited amount of time was available to perform variations on the model. Only a few could be tried; others should be tested--and perhaps new demand or supply relationships developed--before the results are demonstrably useful. Perhaps even at this stage, however, one may hope to discover one or two important clues about potential market behavior.

VARIATIONS

All seven variations performed on the model assume the same demand parameters. The principal exogenous growth factors affecting consumption are shown below, along with their projected annual percentage growth rates, which are held constant throughout the simulation period, 1973 through 1995:[1]

Exogenous Factors	*Growth Rates*
Gross national product	3.88
Value added in the industrial sector	3.50
Commercial-sector floor space[2]	4.50-5.20
Personal income per household	2.24
Personal income per capita	2.91
Number of households	1.60
Population	0.94

These figures are based largely on Bureau-of-the-Census demographic projections and personal-income projections by state and region from the *Survey of Current Business*.

The variations fall into two major groups, according to the conditions assumed for the supply of gas, oil, coal, and electricity--a base-case group and a pessimistic-case group. The parameter values chosen for the latter impose lower short-run elasticities of supply, higher (and in some sectors more steeply sloped) LRMC curves, and

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[1] Appendix B gives a complete listing of parameter values.

[2] The assumed rate varies from one region to another.

lower levels for total gas and oil resource availability than in the base case. The maximum feasible growth rates for electricity as a whole, for nuclear power, and for continental U.S. gas production are cut back, and the coefficient for LNG imports is halved. To preserve the viability of market equilibrium, the maximum feasible growth rates for eastern and western coal are given larger values in the early years than they are in the base case.[1] Although coal production is permitted to expand more freely, however, it must still move along higher LRMC curves. Appendix B fully documents the differences between the base and pessimistic cases.

Within each group of cases, two types of variations are tried. The first type varies the delivered prices of imported crude. Variations of 5, 9, and 11 dollars per barrel are used for both the pessimistic and the base cases. The second type is a change in the pricing of utility gas and electricity. As described above in the subsection "Pricing," the "rate-reform" variations assign a zero weight to historical costs, in contrast to the others, which give gradually decreasing emphasis to historical costs.

AGGREGATE ENERGY

Table 1 summarizes the results for aggregate energy across the seven variations. Total energy is measured according to the conventions used by the Bureau of Mines for calculating "gross consumption of energy resources." The total energy bill is the total cost of energy--including transportation, distribution, and taxes--to energy users. Energy imports (measured net of exports) include crude oil, refined petroleum products, pipeline gas, and LNG. (The United States is assumed to remain a net

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[1] In terms of quantities, the effect of loosening constraints for coal is minimal. Coal production in the pessimistic-case variations generally ranges from 5 to 15 percent more than in the base-case variations. In only two instances does it exceed base-case production by more than 15 percent. The principal effect is to avoid a severe disequilibrium in the market for industrial coal that would otherwise drive the price for a ton of coal to a value of well over $1000, given the supply and demand elasticities assumed.

Table 1

ENERGY CONSUMPTION AND IMPORTS, SELECTED YEARS--1973-1995

Variable	Initial Value						
Base year, 1973							
Total energy	75.6 (10^{15} Btu)						
Cost of energy	110.4 (10^{9} $)						
Unit cost of energy	1.46 ($/$10^{6}$ Btu)						
Energy imports	12.5 (10^{15} Btu)						
Cost of imports	8.8 (10^{9} $)						
	Projected Value						
	Base Case[a]				Pessimistic Case[a]		
	$5	$9	$11	$9/RR	$5	$11	$9/RR
1980							
Total energy	96.1	93.9	93.0	89.5	93.3	91.2	87.9
Cost of energy	163.3	184.8	194.2	192.2	184.3	197.3	157.4
Unit cost of energy	1.70	1.97	2.09	2.15	1.98	2.16	2.25
Energy imports	13.3	6.5	4.7	7.6	10.0	6.4	9.5
Cost of imports	13.9	11.7	9.9	13.2	15.1	13.0	15.7
1985							
Total energy	110.4	109.9	109.3	103.6	106.3	104.8	99.6
Cost of energy	204.2	219.6	228.1	229.5	225.0	232.6	239.4
Unit cost of energy	1.85	2.00	2.09	2.22	2.12	2.22	2.40
Energy imports	17.6	6.5	3.8	8.6	10.7	6.5	12.4
Cost of imports	21.0	12.4	8.6	15.5	18.0	13.3	20.6
1995							
Total energy	140.5	139.2	137.9	133.4	134.0	133.5	127.5
Cost of energy	320.8	330.0	342.7	339.0	350.8	352.3	362.1
Unit cost of energy	2.28	2.37	2.49	2.54	2.62	2.64	2.84
Energy imports	26.3	15.1	10.1	18.1	19.7	16.3	23.4
Cost of imports	39.2	27.5	22.0	31.8	38.6	33.1	45.5

[a]Prices are for one barrel of crude. "RR" stands for "rate reform."

exporter of coal and electricity. These exports are included in the residual demand categories.) The total cost of imports is the delivered cost, p.o.e.

The figures in the table show that between 1973 and 1985 total energy use increases by a factor ranging from 32 to 46 percent, corresponding to average annual growth rates of 2.3 to 3.2 percent. Since the model incorporates a real GNP growth rate of nearly 3.9 percent, these values imply an annual rate of decline of 0.7 to 1.5 percent in the energy/GNP ratio. This contrasts with a nearly zero trend rate of decline, as indicated by weighted averages for the years 1951 to 1953 and 1971 to 1973, and reflects both higher-than-historic energy prices and low estimated elasticities of demand with respect to income or output in some sectors.

During the period 1973 through 1985, the decline in the projected energy growth rate, observed as one moves from left to right within each case in the table, is clearly related to the increasing cost of energy. In 1980, the average cost of one million Btu rises from \$1.70 to \$2.25; in 1985, from \$1.85 to \$2.40. In the period 1985 through 1995, all cases have average annual growth rates of 2.1 to 2.5 percent per year. Despite the lack of important differences in growth rates, energy use, *in toto*, continues to decline across cases in response to higher energy prices. Table 2 documents this decline with some illustrative arc elasticities of use with respect to price which indicate that elasticity increases as the percentage change in price increases,[1] and as more time is allowed for adjustment.[2] These calculations suggest that the long-run price elasticity of energy as a whole may be quite low, though not without quantitative significance, for the sorts of price variations simulated in the model.

The drop in elasticity between 1980 and 1985 in row 1 of Table 2, which compares the two base-case runs, is due to the relatively large medium-run supply elasticity for domestic coal and gas. By 1985, these industries will have been able to expand their production considerably

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[1] Compare the results within each column, moving from top to bottom.

[2] Compare the results within each row, moving from left to right.

Table 2

ARC ELASTICITIES OF TOTAL ENERGY WITH RESPECT TO PRICE

Cases	% Increase in Unit Cost of Energy			% Decline in Total Energy			Arc Elasticity		
	1980	1985	1995	1980	1985	1995	1980	1985	1995
Base, $5, vs. base, $11	22.9	13.0	9.2	-3.23	-1.00	-1.85	0.14	1.08	0.20
Base, $5, vs. pessimistic, $11	27.1	20.0	15.8	-5.10	-5.07	-4.98	0.19	0.25	0.32
Base, $5, vs. pessimistic, $5, plus rate reform	32.4	29.7	24.6	-8.53	-9.78	-9.25	0.26	0.33	0.38

in response to the increased demand for their products due to the increase in oil prices.[1] By the 1990s, however, domestic reserves are getting low and the previous rates of supply expansion cannot be retained, so the total energy elasticity is larger again. In the pessimistic-case simulations, domestic supply is never responsive enough to cause such a low total elasticity.

IMPORTS

Referring again to Table 1, the model predicts, perhaps surprisingly, a declining level of energy imports through 1980 in all except the $5/bbl variation of the base case; even in that instance, imports rise by a rather small amount. Through 1985, imports continue to be restrained; in 1995, they still barely exceed the 1973 level in several cases. Despite the relatively subdued growth projected for energy imports, the energy import bill is from 2.5 to 5.2 times greater than in 1973--demonstrating that, even if energy-import growth is low in physical terms, such imports may substantially affect exchange rates, the balance of trade, and the balance of payments.

But why the limited growth in imports--specifically, oil imports? (Oil imports account for most of the difference in import levels between any two variations.) The response of demand to price is clearly important. Comparisons across cases show that increases in crude oil prices reduce demand outlays on imports as well. Simulated demand for energy imports has a long-run elasticity with respect to price of more than one. If this figure accurately reflects U.S. market conditions, it is doubtful that imported oil can be permanently priced at a level much above the LRMC of continental U.S. production. In fact, some pressure should exist to undercut U.S. production with less-than-LRMC prices; but whether domestic oil producers and, in turn, the federal government would allow this situation to occur is debatable in view of the relative ease with which producers have succeeded in obtaining protection from imports in the past via quotas and tariffs. If import

[1] See Table 3 and more detailed discussion below, pp. 30-32.

prices do closely match the LRMC of continental U.S. crude production, the simulation results suggest that oil-import growth will more or less match the growth of total energy use.[1]

RATE REFORM

The rate-reform variations in Table 1 show that a gas/electricity ratio approximating marginal costs more closely would measurably reduce consumption and add perhaps $7.5 to $10 billion to the nation's energy bill. Oil imports would also rise by 20 to 30 percent, if the variations are suitably representative of reality. Table 3, which records energy supply by type, provides additional details about the effects of rate reform on the energy-source mix. Most notable is the decline in gas--13 to 23 percent in the $9/bbl base case. Secondarily, one can observe declines in the energy sources used predominantly or entirely for electricity generation--coal, nuclear, and "other"--and an increase in oil.

SHARES

As regards the projected evaluation of energy-supply shares, Table 3 holds few surprises. Nuclear power increases the most, rising from less than 1 percent in 1973 to 10 to 11 percent in 1985, and to roughly 20 percent by 1995 in the base-case variations, primarily as a result of the exogenously given increase in the share of electricity generation from nuclear power plants. Of the three principal fossil fuels, oil decreases the least by 1995. Several variations, in fact, project slight increases--perhaps reflecting the limited fuel-substitution possibilities in the transportation sector, where petroleum products account for the great bulk of energy consumed. The shares of coal and gas decline notably by 1995, the former mainly because of reduced electricity generation by coal. The latter declines in part because of reduced electricity generation by gas; but other influences,

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[1] In all variations of the model, considerable amounts of crude become available from Alaska--over 5 X 10^{15} Btu by 1980; in some variations, small amounts of shale oil and syncrude begin to appear in 1980. More pessimistic assumptions regarding the availability of energy from these sources would obviously cause increased imports.

Table 3

ENERGY SUPPLY[a] BY TYPE, SELECTED YEARS--1973-1995
(10^{15} Btu)

Energy Type	Initial Value						
Base year, 1973							
Coal	14.9						
Gas[b]	24.2						
Oil[c]	33.0						
Nuclear[d]	0.7						
Other[e]	2.8						
	Projected Value						
	Base Case[f]				Pessimistic Case[f]		
	$5	$9	$11	$9/RR	$5	$11	$9/RR
1980							
Coal	19.5	20.2	20.4	19.7	22.4	22.7	20.9
Gas	29.0	30.6	30.9	26.5	27.7	28.2	24.6
Oil	40.5	35.7	34.1	36.6	37.5	34.5	37.1
Nuclear	4.4	4.6	4.7	4.3	2.9	3.0	2.7
Other	2.6	2.7	2.8	2.5	2.8	2.8	2.5
1985							
Coal	20.4	23.2	23.7	22.2	25.0	25.7	23.4
Gas	29.7	32.5	33.4	26.3	30.2	31.1	24.2
Oil	46.7	40.1	37.8	42.0	41.7	38.5	43.3
Nuclear	11.1	11.6	11.8	10.8	6.7	6.9	6.2
Other	2.4	2.5	2.5	2.3	2.6	2.7	2.4
1995							
Coal	18.3	19.7	20.1	19.6	22.7	23.1	22.0
Gas	33.1	32.8	34.4	25.2	32.2	33.3	24.3
Oil	62.9	56.6	52.9	59.8	56.3	54.1	60.0
Nuclear	27.1	27.9	28.3	26.8	20.4	20.7	19.0
Other	2.1	2.1	2.1	2.1	2.3	2.3	2.1

[a]Includes imports.

[b]Excludes natural gas liquids (NGL).

[c]Includes NGL.

[d]At 10,000 Btu/kwh.

[e]Equals "other electricity" converted at 10,000 Btu/kwh.

[f]"RR" stands for "rate reform."

difficult to identify individually, also appear to be at work. The low value assumed for the elasticity of industrial gas and oil demand with respect to output (0.33) may be a factor. The industrial sector is the largest gas-consuming sector, and its demand increase exerts considerable leverage on the total.

Table 4 demonstrates the evolution of generation shares. The nuclear share rises from 4 to 22 to 35 percent in 1985 and 50 to 64 percent in 1995. The gas and oil shares decline to roughly half their 1973 values by 1995. Coal fills most of the gap left by nuclear in the pessimistic case, but its share still declines from 46 to 30 percent by 1995 in that case. In the base case, it declines even farther to 19 percent. These share trends directly reflect the generation-allocation equations described in Sec. II.

Table 4

APPROXIMATE GENERATION SHARES,[a] SELECTED YEARS--
1973-1995 (Percent)

Fuel	1973	Base		Pessimistic	
		1985	1995	1985	1995
Coal	46	39	19	49	30
Gas and oil	26	18	12	21	14
Nuclear	4	35	64	22	50
Other	15	8	5	8	6

[a]Shares may not sum to 100 owing to rounding errors.

CONSUMPTION BY USER SECTOR

Table 5 summarizes energy consumption by user sector. The figures reported in the table include electricity (net of transmission and distribution losses) converted at 3412 Btu/kwh. The industrial sector totals include small amounts of coal delivered to the residential sector or exported. The "other" category includes transmission and distribution losses in the gas- and oil-supply sectors, net additions to fuel stocks, the unallocated portions of nominal commercial gas and oil use,

consumption of nonpetroleum energy by the transportation sector, military air transportation energy use, and other minor items, The minimum figures in the table correspond to either $11/bbl variations of both cases or the rate-reform variation of the pessimistic case. The maximum figures are all taken from the $5/bbl variation of the base case.

Table 5

ENERGY CONSUMPTION BY SECTOR, SELECTED YEARS--1973-1995
(10^{15} Btu)

Case	Residential	Commercial	Industrial	Transportation	Other
1973	10.30	2.94	25.28	16.62	7.00
1980					
Minimum	11.22	3.75	27.08	19.93	7.51
Maximum	12.42	3.91	29.93	21.84	8.44
1985					
Minimum	12.66	4.55	27.87	29.09	8.17
Maximum	14.38	4.80	33.05	26.26	9.29
1995					
Minimum	16.76	6.97	26.68	36.15	10.08
Maximum	18.87	7.24	33.78	38.29	11.18

The table reveals a pattern of widely differing growth rates for energy consumption. Commercial demand, owing primarily to the high projected rates of growth of commercial floor space, increases the most between 1973 and 1995, ranging from 137 to 146 percent. Close behind is the transportation sector, in which energy use increases from 118 to 130 percent. A nearly five-fold expansion of energy use for civilian air transportation is an important factor in this growth, as is a projected 126 to 133 percent increase in the stock of motor vehicles.[1] The growth of residential demand is more subdued, ranging

[1]The latter corresponds to a rise from 1.46 to 2.29 to 2.36 automobiles per household, despite a decline of 14 percent in average household size--from 3.11 in 1972 to 2.66 in 1995. These figures imply a

from 63 to 83 percent. Since population increases by only 23 percent and households by 44 percent, the residential demand projections imply continued increases in energy use per capita and per household. The industrial sector is the largest of the five in 1973, but it grows most slowly and winds up second to transportation in 1995. Consumption increases only 13 to 34 percent for the industrial sector, despite a projected increase of real industrial output of 121 percent. Two factors contribute heavily to the 2.7 percent per year decline projected for industrial energy intensity--the estimated 3.5 percent per year negative time-trend factor in the coal demand equation, and the low estimated long-run elasticities of demand with respect to output for gas/oil (0.33) and electricity (0.64).[1]

In contrast to the extreme differences in projected demand growth rates by sector, the differences in sectoral growth rates and relative sectoral demand levels induced by the seven variations are rather small. The former are due primarily to income- or output-related terms in the demand equations and the latter to supply assumptions operating through the price terms in the demand equations, which, as Table 2 indicates, lead to only modest consumption adjustments.

PRICES AND COSTS

The projected movements of market prices and marginal costs of energy are reported in Tables 6 and 7, respectively. Looking first at the table of prices, one observes an irregular pattern of price movements across energy types and model variations, though a general upward trend is evident. An exception to this trend occurs in the case of industrial coal, which undergoes a period of extremely tight demand-

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saturation of 0.88 automobiles per person; and since about one in four persons would be too young to drive in 1995, they further imply a saturation of 1.17 automobiles per person over 17 years of age, a decidedly improbable result, though not an impossible one if increasing affluence induces individuals to buy different vehicles for different purposes.

[1] If the coal time-trend factor is halved and the long-run output elasticities for gas/oil and electricity are all raised to unity, industrial demand rises from 32.0 to 45.6 10^{15} Btu for 1995 in the \$11 variation of the base case, a difference of 42 percent. Total energy consumption increases by 12 percent.

Table 6

MARKET PRICES, SELECTED YEARS--1973-1995

Prices	Industrial Coal ($/Ton)	Electricity (mills/kwh)		Gas ($/$10^6$ Btu)		Oil ($/$10^6$ Btu)		Regular Gasoline (¢/gal)
		Residential	Industrial	Residential	Industrial	Residential	Industrial	
1973	13.04	24.1	11.6	1.19	0.45	1.59	0.85	39.4
1980								
Base								
$5	16.42	26.4	14.7	1.39	0.65	1.78	1.04	41.9
$9	26.13	26.6	14.9	1.50	0.76	2.50	1.76	50.9
$11	30.64	26.6	14.9	1.54	0.80	2.86	2.12	55.3
$9/RR	18.01	30.0	18.6	1.67	0.92	2.50	1.76	50.9
Pessimistic								
$5	16.17	28.6	16.8	1.57	0.83	2.34	1.60	48.8
$11	18.00	29.1	17.2	1.64	0.90	2.86	2.12	55.3
$9/RR	13.22	33.6	22.0	1.79	1.05	2.50	1.76	50.9
1985								
Base								
$5	11.31	28.6	16.4	1.50	0.76	1.92	1.18	43.6
$9	11.29	28.7	16.5	1.54	0.79	2.50	1.76	50.9
$11	11.28	28.8	16.6	1.54	0.80	2.86	2.12	55.3
$9/RR	11.31	31.8	19.4	1.81	1.07	2.50	1.76	50.9
Pessimistic								
$5	13.25	31.7	19.3	1.57	0.83	2.48	1.74	50.6
$11	13.25	31.8	19.4	1.58	0.84	2.86	2.12	55.3
$9/RR	13.24	36.8	24.2	1.94	1.20	2.50	1.76	50.9
1995								
Base								
$5	11.35	34.8	20.9	1.77	1.03	2.23	1.49	47.5
$9	11.35	34.9	21.0	1.82	1.08	2.50	1.76	50.9
$11	11.35	34.9	21.0	1.84	1.10	2.86	2.12	55.3
$9/RR	11.37	37.6	23.2	2.05	1.30	2.50	1.76	50.9
Pessimistic								
$5	13.35	38.9	24.7	1.91	1.16	2.78	2.04	54.4
$11	13.36	38.9	24.8	1.92	1.18	2.86	2.12	55.3
$9/RR	13.33	42.7	28.1	2.20	1.45	2.50	1.76	50.9

NOTE: "RR" stands for "rate reform."

Table 7

LONG-RUN MARGINAL COSTS, SELECTED YEARS--1973-1995

Prices	Coal ($/Ton)		Gas ($/$10^6$ Btu)			Oil ($/bbl)			
	East	West	Continental U.S.	Alaska	Synthetic	Continental U.S.	Shale	Alaska	Synthetic
1973	8.03	3.00	0.42	0.80	n.a.	3.88	n.a.	3.02	n.a.
1980									
Base									
$5	8.29	3.00	0.57	0.80	1.40	4.91	5.98	3.02	6.67
Pessimistic									
$5	10.30	4.17	0.72	0.80	1.55	8.08	5.98	4.02	7.42
1985									
Base									
$5	8.51	3.00	0.68	0.80	1.31	5.73	5.98	3.02	6.25
Pessimistic									
$5	10.54	4.40	0.87	0.80	1.49	8.88	5.98	4.02	7.12
1995									
Base									
$5	8.87	3.00	0.88	0.80	1.09	7.50	5.98	3.02	6.40
Pessimistic									
$5	10.99	5.00	1.13	0.80	1.28	10.58	5.98	4.02	7.31

supply conditions in the late 1970s in most of the variations. In the $9/bbl, $11/bbl, and $9/bbl-plus-rate-reform variations of the base case, the price of industrial coal peaks at values of $33.90, $41.30, and $20.69 per ton in 1977. Thereafter, the market moves quickly toward a long-run equilibrium position. By 1985, coal is available at prices below or only slightly above those for 1973.

This price pattern results from the increase in the price of oil causing an increase in the demand for other fuels. The large initial increases in the price of coal work themselves out after several years. As the price of coal increases with demand, coal producers begin to expand their operations. By 1985, supply has expanded sufficiently to cover demand at reasonably stable long-run prices. In the rate-reform variations, the pressures on coal are somewhat reduced, since increases in coal prices are more quickly reflected in electric utility rates and thence in the demand for coal by electric utilities.

Electric and gas utility rates climb steadily over time and are greater in the pessimistic than in the base case, and in the rate-reform than in the conventional variations. In 1995, residential rates have increased by 44 to 77 percent (electricity) and 48 to 85 percent (gas) over their 1973 values. Far more spectacular percentage increases occur in industrial rates, owing to the relatively large share of the price represented by raw energy costs (as opposed to transmission and distribution costs). Electricity rates rise by 80 to 142 percent and gas rates by 129 to 222 percent.

Petroleum product prices are keyed directly to the underlying assumptions about crude oil prices, and are therefore lowest in the $5/bbl variations and highest in the $11/bbl variations. As in the case of utility rates, the residential fuel-oil price rises less than the average price of industrial oil: 40 to 80 percent for the former, compared to 75 to 149 percent for the latter by 1995. Owing to the large distribution and taxation component in total cost, the relative increase in the price of regular gasoline is the smallest of those recorded in the model. It ranges from 22 to 40 percent by 1995.

Table 7 lists the underlying LRMCs for fuel at the point of production or the port of entry into the continental United States. Values

are shown for only the $5 variations of the two case groupings, since the differences within groups are very slight. The LRMCs for Alaskan gas and oil shale are constant by assumption, and that for Alaskan crude oil is invariant within each of the groups. The LRMC of western U.S. coal is very nearly constant in the base case, because of the exponential LRMC curve described in Sec. II. In the pessimistic case, the LRMC curve is linear with respect to depletion. The LRMC curves for continental U.S. gas and oil production are also linear. These curves are responsible for the only notable increases in LRMC. The LRMCs for synthetic fuels have two components, one dependent on coal prices and one fixed by assumption. In the syngas industry, increases in coal costs are offset by an assumed reduction in noncoal costs. Nevertheless, in most variations of the model, a syngas industry fails to develop owing to its high LRMC. Oil shale and syncrude make modest contributions to supply by 1995. Alaskan gas plays a small or negligible role in the base-case variations, owing to its high LRMC. In the pessimistic-case variations, it plays a greater role, since the LRMC of continental U.S. production exceeds that of Alaska as early as 1983.

RESOURCE DEPLETION

The cumulative development of oil and gas reserves is summarized in Table 8 for gas and oil, as of 1995. These figures document the low rate of development projected for Alaskan gas and the high rate projected for Alaskan oil,[1] a rate sufficiently great to produce a higher cumulative total, relative to the nominal maximum, for Alaskan oil than for continental U.S. oil, despite the considerably shorter period of exploitation in Alaska. Tha maximum feasible growth rates associated with these relative depletion levels illustrate the sensitivity of these rates to cumulative resource development.

CONCLUSIONS

It would be unwise to hazard any definite assertions about energy market behavior on the basis of the small set of preliminary results

[1] Up to 6.2 million barrels per day by 1995.

Table 8

RESOURCE DEPLETION, JANUARY 1, 1995--GAS AND OIL

Item	Base Case		Pessimistic Case	
	$5	$9/RR	$5	$9/RR
Cumulative additions to reserves (10^{15} Btu)				
Gas				
Continental U.S.	685	592	630	543
Alaska	5	3	96	38
Oil				
Continental U.S.	471	539	451	461
Alaska	231	241	208	208
As percent of nominal maximum				
Gas				
Continental U.S.	44	38	70	60
Alaska	1	1	38	15
Oil				
Continental U.S.	52	60	60	61
Alaska	59	62	65	65
Maximum feasible growth rate (%)				
Gas				
Continental U.S.	1.5	1.5	-0.4	0.4
Alaska	50.0	15.0	5.9	22.0
Oil				
Continental U.S.	1.0	0.4	0.4	0.3
Alaska	2.0	1.3	0.6	0.5

Note: "RR" stands for "rate reform."

described above. A number of model characteristics must be studied further for verification:

- The price elasticity of oil imports, which is greater than unity.
- The relative inelasticity of total energy consumption with respect to price.
- The supersaturation of household vehicle ownership by 1995.

Many other model characteristics are probably sensitive to parameters that were not varied in this first exercise. Continuing experimentation with the model may expose them.

Appendix A

MATHEMATICAL STATEMENT OF THE MODEL

This appendix presents the mathematical statement of the model. First, the definitions of the variables that are used in the model, as well as their subscripts and superscripts, are given. Then the basic equations of the model are listed.

The model is best described as consisting of two parts--a "preiterative" and an "iterative" section. The preiterative section sets up the basic structure of energy supply (and a few elements of demand) each year. The iterative section then solves for the market-clearing prices and quantities of energy. For ease in programming as well as understanding the model, each of these sections has its own "class" of endogenous variables.

Each year, the model's representation of the structure of the energy sectors of the U.S. economy is updated. The variables associated with this basic structure are classed (and programmed) as the "Z" variables. Values of these variables must be determined only once each year, in the preiterative section. Then, based on the current values of the Z variables, the model solves for the equilibrium prices and quantities of energy in the iterative section. The prices and quantities (and several other variables) determined there are known as the "Y" variables. They are the current energy prices and the other endogenous variables directly dependent on current energy prices. The Z's are the endogenous variables independent of current energy prices.

The equations for the preiterative portion of the model appear first, followed by those for the iterative portion. Within each section, the arrangement of the equations is roughly as follows:

Supply equations
- Coal
- Electricity
- Natural Gas
- Petroleum

Demand equations
- Residential
- Industrial
- Commercial
- Transportation

Market clearing

Finally, one equation representing a "postiterative" part of the model is presented.

DEFINITIONS

Variables

Q = domestic production,

$\bar{Q}$ = *ex-ante* full-capacity output level,

$\tilde{Q}$ = *ex-post* full-capacity output level,

k = long-run marginal cost (LRMC),

w = weighting factor,

c = short-run marginal cost (SRMC) or (sometimes) price,

R = cumulative gross additions to reserves as of January 1,

R^* = maximum value of R permitted,

r = maximum permissible growth rate along the LRMC curve,

ℓ = loss factor,

A = square feet of commercial floor space,

p = price,

s = share of incremental floor space,

b = energy-use coefficient for commercial floor space (five regions, three system configurations),

D = actual demand,

D^* = target demand,

GM = gasoline mileage,

MV = current stock of motor vehicles/initial stock,

VH = motor vehicles per household,

T = tax rate,

M = imports,

F = ratio (of two quantities indicated in subscript),

G = demand-supply gap,

I = iteration step factor,

m,n = coefficients in commercial-sector trade-off equations.

Subscripts

t = time,
c = coal,
e = electricity,
g = gas,
p = crude petroleum,
r = refined petroleum products (oil),
eu = electric utility sector,
u = uranium,
nf = nuclear fuel,
o = initial (t=o),
lng = liquified natural gas,
alt = "alternative" fuels (gas and oil)
syn = synfuel,
y = real household income or economic activity generally,
h = household(s),
g/r = gas and oil
v = real value added by industry,
yc = real per-capita income,
pop = population,
gnp = real gross national product,
a = floor area
f = fossil fuels,
mv = motor vehicles,
gm = gasoline mileage.

Superscripts[1]

w = west,
e = east,
k = long-run marginal cost (LRMC),
oth = all other,
n = nuclear power,
ala = Alaska,
us = continental United States,
m = imports,
min = minimum
s = shale,
res = residential,
com = commercial,
ind = industrial,
we = west to east,
air = civilian air transport,
rr = railroads,
pp = passenger,
f = freight,
pl = pipelines,
ww = waterways,
w/pl = waterways and pipelines,
sg = syngas,
sp = syncrude.

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[1] When different from subscripts.

EQUATIONS

The Preiterative Section

Z(1),Z(2) $$\bar{Q}^i_{ct} = \max\begin{cases}(1+\pi^i_c)\cdot[\bar{Q}^i_{ct-1} + \rho^i_c(Q^i_{ct-1} - \bar{Q}^i_{ct-1})] \\ (1-\delta_c)\cdot \bar{Q}^i_{ct-1}\end{cases}, \quad i = e,w$$

Z(3),Z(4) $$k^i_{ct} = k^i_{co} + \beta^i_c \left[\sum_{j=o}^{t-1}\left(\frac{Q^i_{cj}}{Q^{i,max}_c}\right)\right]^{\eta^i_c}, \quad i=e,w$$

Z(5) $$w^w_{ct} = Q^w_{ct-1}/(Q^w_{ct-1} + Q^e_{ct-1})$$

Z(6) $$\bar{Q}_{et} = \max\begin{cases}(1+\pi_e)\bar{Q}_{et-1} \\ (1-\delta_e)\bar{Q}_{et-1}\end{cases}$$

Z(7),Z(8) $$k^i_{et} = \underline{k}^i_e \cdot \left(\frac{1}{1+\rho_e}\right)^t + \bar{k}^i_e \cdot \left[1 - \left(\frac{1}{1+\rho_e}\right)^t\right], \quad i=c,n$$

Z(9) $$w_{ct} = Q^c_{et-1}/(Q^c_{et-1} + Q^n_{et-1})$$

Z(10) $$w_{et} = \gamma^t_e \cdot (Q_{eo}/Q_{et-1})$$

Z(11) $$k_{ut} = \max\begin{cases}k_{uo} \\ \beta^k_u \cdot \sum_{i=o}^{t-1} Q_{ui}\end{cases}$$

Z(12)-Z(14) $$\bar{Q}^i_{et} = \bar{Q}^i_{eo} + \delta^i_e \cdot t \ , \quad i = r, g, oth$$

Z(15) $$c_{nf,t} = c_{nf,o}\left(\frac{1}{1+\rho_e}\right)^t + c_{nf,\max}\left[1 - \left(\frac{1}{1+\rho_e}\right)^t\right]$$

Z(16) $$\bar{Q}^{n}_{et} = \left\{\sigma^{n,max}_{e} / [1 + \beta^{n}_{e} \cdot (\gamma^{n}_{e})^{t}]\right\} \bar{Q}_{et}$$

Z(145) $$\bar{Q}^{c}_{et} = \bar{Q}_{et} - \bar{Q}^{n}_{et} - \bar{Q}^{r}_{et} - \bar{Q}^{g}_{et} - \bar{Q}^{oth}_{et}$$

Z(17)-Z(18) $$\tilde{Q}^{i}_{gt} = \max \begin{cases} \bar{Q}^{i}_{gt-1} + \rho^{i}_{g}(Q^{i}_{gt-1} - \bar{Q}^{i}_{gt-1}) \\ (1 - \delta_{g})\bar{Q}^{i}_{gt-1} \end{cases} , \quad i = ala, us$$

Z(19),Z(20) $$R^{i}_{gt} = R^{i}_{gt-1} + \nu_{g} \cdot \left[\tilde{Q}^{i}_{gt} - (1 - \delta_{g})\tilde{Q}^{i}_{gt-1}\right] , \quad i = ala, us$$

Z(21),Z(22) $$Q^{i,max}_{gt} = \max \begin{cases} Q^{i,max}_{gt-1} \\ \tilde{Q}^{i}_{gt} \end{cases} , \quad i = ala, us$$

Z(23),Z(24) $$r^{i}_{gt} = \begin{cases} \gamma^{i}_{g} - \gamma^{i}_{g} \cdot (R^{i}_{gt}/R^{i*}_{g}) & , R^{i}_{gt}/R^{i*}_{g} \leq \phi_{g} \\ -\delta_{g} \cdot \left[1 - \dfrac{Q^{i,max}_{gt}}{\tilde{Q}^{i}_{gt}} \cdot \dfrac{1-(R^{i}_{gt}/R^{i*}_{g})}{1 - \phi_{g}}\right] & , R^{i}_{gt}/R^{i*}_{g} > \phi_{g} \end{cases} , \quad i = ala, us$$

Z(25) $$\bar{Q}^{ala}_{gt} = \begin{cases} (1 + r^{ala}_{go})Q^{ala}_{gt-1} & , t \leq 6 \quad \text{or} \quad k^{ala}_{gt-1} > c^{us}_{gt-1} \\ (1 + r^{ala}_{gt})\tilde{Q}^{ala}_{gt} & , \text{otherwise} \end{cases}$$

Z(26) $$\bar{Q}^{us}_{gt} = (1 + r^{us}_{gt})\tilde{Q}^{us}_{gt}$$

Z(27) $$\bar{Q}^c_{gt} = \begin{cases} 0.0 \ , & t < t^c_g \\ \bar{Q}^c_{go} \ , & t = t^c_g \\ (1 + \pi^c_g) \cdot \max \begin{cases} Q^c_{gt-1} \\ \bar{Q}^c_{go} \end{cases} , & t > t^c_g \end{cases}$$

Z(28),Z(29) $$k^i_{gt} = k^i_{go} + \beta^i_g \cdot R^i_{gt} \quad , i = ala, us$$

Z(30) $$k^c_{gt} = \bar{k}^c_{gt}$$

Z(31) $$w_{gt} = (\gamma_g)^t \cdot \left[\left(\sum_{i=ala}^{us,c} Q^i_{go} + \sum_{j=g}^{lng} M_{jo} \right) / \left(\sum_{i=ala}^{us,c} Q^i_{gt-1} + \sum_{j=g}^{lng} M_{jt-1} \right) \right]$$

Z(32) Not used

Z(33) $$c^m_{lng,\,t} = \bar{c}^m_{lng}$$

Z(34),Z(35) $$\tilde{Q}^i_{pt} = \max \begin{cases} \bar{Q}^i_{pt-1} + \rho^i_p (Q^i_{pt-1} - \bar{Q}^i_{pt-1}) \\ (1 - \delta_p) \tilde{Q}^i_{pt-1} \end{cases} , i = ala, us$$

Z(36),Z(37) $$\ell^i_{pt} = \begin{cases} \lambda^{max}_p \ , & Q^i_{pt-1} > \tilde{Q}^i_{pt} \\ -\lambda^{min}_p \ , & Q^i_{pt-1} < \tilde{Q}^i_{pt} \end{cases} , i = ala, us$$

Z(38),Z(39) $$R^i_{pt} = R^i_{pt-1} + \nu_p \cdot [\tilde{Q}^i_{pt} - (1 - \delta_p)\tilde{Q}^i_{pt-1} + \ell^i_{pt} \cdot (Q^i_{pt-1} - \tilde{Q}^i_{pt})] \quad , i = ala, us$$

Z(40), Z(41) $$Q_{pt}^{i,max} = \max \begin{cases} Q_{pt-1}^{i,max} \\ \gamma^i Q_{pt}^{i} \end{cases}, i = ala, us$$

Z(42), Z(43) $$r_{pt}^{i} = \begin{cases} \pi_p^i - \gamma_p^i \cdot (R_{pt}^i / R_p^{i*}), & , R_{pt}^i / R_p^{i*} \leq \phi_p, \\ -\delta_p \cdot \left[1 - \dfrac{Q_{pt}^{i,max}}{\bar{Q}_{pt}^i} \cdot \dfrac{1 - (R_{pt}^i / R_p^{i*})}{1 - \phi_p} \right], & R_{pt}^i / R_p^{i*} > \phi_p \end{cases}$$

$$i = ala, us$$

Z(44),Z(45) $$\bar{Q}_{pt}^i = (1 + r_{pt}^i)\, \gamma^i Q_{pt}^i \quad , i = ala, us$$

Z(46), Z(47) $$\bar{Q}_{pt}^i = \begin{cases} 0.0, & t < t_i \\ \bar{Q}_{po}^i, & t = t_i \\ (1 + \pi_p^i) \cdot \max \begin{cases} Q_{pt-1}^i \\ \bar{Q}_{po}^i \end{cases} & t > t_i \end{cases} \quad , i = c, s$$

Z(48) $$R_{pt}^s = R_{pt-1}^s + Q_{pt-1}^s$$

Z(49)-Z(51) $$k_{pt}^i = k_{po}^i + \beta_p^i \cdot R_{pt}^i \quad , i = ala, us, s$$

Z(52) $$k_{pt}^c = \bar{k}_{pt}^c$$

Z(53) $$c_{rt} = \bar{c}_{rt}$$

Z(54)-Z(58) $A^i_t = A^i_o\ (1 + \pi^i_a)^{(t+2)}$, $i = 1, \ldots, 5$

Z(59)-Z(73) $p^i_{jt-1} = p^{com}_{jt-1} + \alpha^i_j$, $i = 1, \ldots, 5$; $j = g, r, e$

Z(74)-Z(78) $p^i_{ft-1} = \min \begin{cases} p^i_{gt-1} \\ p^i_{rt-1} \end{cases}$, $i = 1, \ldots, 5$

Z(79)-Z(83) $s^i_{1t} = \begin{cases} \bar{s}^i_1 & \text{if } p^i_{ft-1} > m_1 + n_1\, p^i_{et-1}\,, \\ \underline{s}^i_1 & \text{otherwise} \end{cases}$ $i = 1, \ldots, 5$

Z(84)-Z(88) $s^i_{2t} = \begin{cases} \bar{s}^i_2 & \text{if } p^i_{ft-1} \begin{cases} \leq m_1 + n_1\, p^i_{et-1} \\ \geq m_2 + n_2\, p^i_{et-1}, \end{cases} i = 1, \ldots, 5 \\ \underline{s}^i_2 & \text{otherwise} \end{cases}$

Z(89)-Z(93) $s^i_{3t} = \begin{cases} \bar{s}^i_3 & \text{if } p^i_{ft-1} < m_2 + n_2\, p^i_{et-1},\ i = 1, \ldots, 5 \\ \underline{s}^i_3 & \text{otherwise} \end{cases}$

Z(94)-Z(108) $b^i_{gkt} = \beta^i_{fk} / [1 + (p^i_{gt-1}/p^i_{rt-1})^{\sigma^i}]$, $i = 1, \ldots, 5$; $k = 1, \ldots, 3$

Z(109)-Z(123) $b^i_{rkt} = \beta^i_{fk} - b^i_{gkt}$, $i = 1, \ldots, 5$; $k = 1, \ldots, 3$

Z(124)-Z(138) $b^i_{ekt} = \beta^i_{ek}$, $i = 1, \ldots, 5$; $k = 1, \ldots, 3$

Z(139)-Z(141) $$D_{jt}^{com,*} = \sum_{i=1}^{5} \left[\left(A_t^i - \rho^{com} A_{t-1}^i \right) \cdot \left(\sum_{k=1}^{3} b_{jkt}^i \; s_{kt}^i \right) \right]$$

$$j = g,r,e$$

Z(142) $$GM_t = \left(\frac{p_{rt-1}^{mv}}{p_{ro}^{mv}} \right)^{\eta_r^{gm}}$$

Z(143) $$MV_t = \left[\eta_o^{mv} + \eta_y^{mv} \; (1 + \pi_y)^t + \eta_r^{mv} \left(\frac{p_{r,t-1}^{mv}}{p_o^{mv}} \right) \right] \cdot \frac{(1 + \pi_h)^t}{VH_o} + \eta^{mv} \cdot MV_{t-1}$$

Z(144) $$w_{rt}^{ind} = D_{rt-1}^{ind} \; / \; (D_{rt-1}^{ind} + D_{gt-1}^{ind})$$

The Iterative Section

Y(1) $$p^{ind}_{ct} = \max \begin{cases} p^{ind}_{ct} \text{ (last iteration)} \cdot (1 + T_1) \\ k^{e}_{ct} + \tau^{ind}_{c} - \phi^{w}_{c} \cdot w^{w}_{ct} \cdot \tau^{we}_{c} \end{cases}$$

Y(12) $$c^{us}_{gt} = \max \begin{cases} c^{us}_{gt} \text{ (last iteration)} \cdot (1 + T_2) \\ k^{us}_{gt} \end{cases}$$

Y(3) $$P_{pt} = \max \begin{cases} \bar{P}_{pt} \\ k^{us}_{pt} \end{cases}$$

Y(4) $$T_{et} = \begin{cases} 0 & , Q_{et} < \bar{Q}_{et} \text{ (last iteration)} \\ T_4 & , Q_{et} = \bar{Q}_{et} \text{ and } T_{et} = 0 \text{ (last iteration)} \\ T_{et} \text{ (last iteration)} \cdot (1 + T_4), & \text{otherwise} \end{cases}$$

Y(5) $$c^{w}_{ct} = p^{ind}_{ct} - \tau^{ind}_{c} - (1 - \phi^{w}_{c} \cdot w^{w}_{ct}) \cdot t^{we}_{c}$$

Y(6) $$c^{e}_{ct} = c^{w}_{ct} + \tau^{we}_{c}$$

Y(7) $$p^{eu}_{ct} = w^{w}_{ct} \cdot \min (c^{w}_{ct}, 1.5\, k^{w}_{ct}) + (1 - w^{w}_{ct}) \cdot \min (c^{e}_{ct}, 1.5\, k^{e}_{ct}) + \tau^{eu}_{e}$$

Y(8) $$k_{et} = w_{ct} (\sigma^{c}_{eu}\, p^{eu}_{ct} + k^{c}_{et}) + (1 - w_{ct}) \cdot \left| c_{nf,t} + \sigma^{n}_{1}\, k_{ut} + \sigma^{n}_{2} (k_{ut} - k_{uo}) + k^{n}_{et} \right|$$

Y(9)-Y(11) $$p^{i}_{et} = \left\{ w_{et} \cdot p^{i}_{eo} + (1 - w_{et}) \left| (1 + \lambda^{i}_{e})\, k_{et} + \tau^{i}_{e} \right| \right\} (1 + T_{et}) \text{ , } i = \text{res, com, ind}$$

Y(13) $$c_{gt}^{ala} = c_{gt}^{us}$$

Y(14) $$c_{gt}^{c} = \begin{cases} c_{gt}^{us} & \text{if } \bar{Q}_{gt}^{c} > 0 \\ 0 & \text{otherwise} \end{cases}$$

Y(15) $$c_{gt}^{m} = \begin{cases} c_{gt}^{us} & \text{if } \mu_{g} > 0 \\ 0 & \text{otherwise} \end{cases}$$

Y(19),Y(20) $$c_{pt}^{i} = \begin{cases} p_{pt} & \text{if } \bar{Q}_{pt}^{i} > 0 \\ 0 & \text{otherwise} \end{cases} \qquad i = c, s$$

Y(21) $$c_{pt}^{m} = p_{pt}$$

Y(22) $$c_{pt}^{us} = p_{pt}$$

Y(23) $$c_{pt}^{ala} = p_{pt}$$

Y(24)-Y(30) $$p_{rt}^{i} = \beta_{r}^{i} (p_{pt} + c_{rt}) + \tau_{r}^{i}, \quad i = \text{res, com, ind, mv, air, rr, oth}$$

Y(31) $$c_{rt}^{m} = p_{pt} + c_{rt}$$

Y(33), Y(34) $$Q_{ct}^{i} = \begin{cases} \bar{Q}_{ct}^{i} \cdot \left[\dfrac{(c_{ct}^{i} / k_{ct}^{i}) - \sigma_{c}^{i}}{1 - \sigma_{c}^{i}} \right]^{\varepsilon_{c}^{i}}, & c_{ct}^{i} \geq k_{ct}^{i}, \\ 0, & \text{otherwise} \end{cases} \qquad i = e, w$$

$$Y(35)\quad Q_{gt}^{ala} = \begin{cases} \bar{Q}_{gt}^{ala} \cdot \left[\dfrac{c_{gt}^{ala}/k_{gt}^{ala} - \sigma_g^{ala}}{1 - \sigma_g^{ala}}\right]^{\varepsilon_g^{ala}}, & c_g^{ala} \geq k_g^{ala} \\ (1 + r_{go}^{ala}) \cdot Q_{gt-1}^{ala}, & \text{otherwise} \end{cases}$$

$$Y(36)\quad Q_{gt}^{us} = \begin{cases} \bar{Q}_{gt}^{us} \cdot \left[\dfrac{c_{gt}^{us}/k_{gt}^{us} - \sigma_g^{us}}{1 - \sigma_g^{us}}\right]^{\varepsilon_g^{us}}, & c_g^{us} \geq k_g^{us} \\ 0, & \text{otherwise} \end{cases}$$

$$Y(37)\quad Q_{gt}^{c} = \begin{cases} \bar{Q}_{gt}^{c} \cdot \left(\dfrac{[c_{gt}^{c} - \beta_g^{c}(\min(c_{ct}^{w}, 1.5\,k_{ct}^{w}) + \tau_c^{g})]/k_{gt}^{c} - \sigma_g^{c}}{1 - \sigma_g^{c}}\right)^{\varepsilon_g^{c}}, \\ \qquad \text{if} \quad c_{gt}^{c} \geq \beta_g^{c}\left[\min(c_{ct}^{w}, 1.5\,k_{ct}^{w}) + \tau_c^{g}\right] + \sigma_g^{c} k_{gt}^{c} \\ \delta_{syn} \cdot Q_{gt-1}^{c}, \text{ otherwise} \end{cases}$$

$$Y(38)\quad Q_{gt} = Q_{gt}^{us} + Q_{gt}^{ala} + Q_{gt}^{c}$$

$$Y(39)\quad M_{gt} = \mu_g \cdot Q_{gt}$$

$$Y(40)\quad M_{lng,t} = \mu_{lng} \cdot Q_{gt}$$

$$Y(2)\quad P_{gt} = \begin{cases} \left[c_{gt}^{us}(Q_{gt} + M_{gt}) + c_{lng,t}^{m} \cdot M_{lng,t}\right] / (Q_{gt} + M_{gt} + M_{lng,t}), \\ \qquad \text{if} \quad c_{gt}^{ala} \geq k_{gt}^{ala} \\ \left[c_{gt}^{us}(Q_{gt} + M_{gt}) + c_{lng,t}^{m} \cdot M_{lng,t} + (\kappa_{gt}^{ala} - c_{gt}^{ala}) \cdot Q_{gt}^{ala}\right] / \\ \qquad (Q_{gt} + M_{gt} + M_{lng,t}), \text{ if } c_{gt}^{ala} < k_{gt}^{ala} \end{cases}$$

Y(16)-Y(18) $$p^i_{gt} = w_{gt} \cdot p^i_{go} + (1 - w_{gt}) \left[(1 + \lambda^i_g)\, p_{gt} + \tau^i_g \right],$$

$$i = res, com, ind$$

Y(32) $$p^{ind}_{alt,t} = p^{ind}_{rt} \cdot w^{ind}_{rt} + p^{ind}_{gt} \cdot (1 - w^{ind}_{rt})$$

Y(41)-Y(42) $$Q^i_{pt} = \begin{cases} \bar{Q}^i_{pt} \cdot \left[\dfrac{c^i_{pt}/k^i_{pt} - \sigma^i_p}{1 - \sigma^i_p} \right]^{\varepsilon^i_p}, & c^i_{pt} \geq k^i_{pt} \\ 0, & \text{otherwise} \end{cases}, \quad i = ala, us$$

Y(43) $$Q^s_{pt} = \begin{cases} \bar{Q}^s_{pt} \cdot \left[\dfrac{c^s_{pt}/k^s_{pt} - \sigma^s_p}{1 - \sigma^s_p} \right]^{\varepsilon_p}, & c^s_{pt} \geq k^s_{pt} \\ \delta_{syn} \cdot \bar{Q}^s_{pt-1}, & \text{otherwise} \end{cases}$$

Y(44) $$Q^c_{pt} = \begin{cases} \bar{Q}^c_{pt} \left(\dfrac{\left[c^c_{pt} - \beta^c_p \left(\min(c^w_{ct},\, 1.5\, k^w_{ct}) + \tau^p_c \right) \right] / k^c_{pt} - \sigma^c_p}{1 - \sigma_p} \right)^{\varepsilon^c_p}, \\ \quad \text{if} \quad c^c_{pt} \geq \beta^c_p \cdot \left[\min(c^w_{ct},\, 1.5\, k^w_{ct}) + \tau^p_c \right] + k^c_p \\ \delta_{syn} \cdot Q^c_{pt-1}, \text{ otherwise} \end{cases}$$

Y(45) $$Q_{pt} = \sum_{i=ala,us}^{c,s} Q^i_{pt}$$

Y(52)-Y(54) $$D^{res}_{it} = D^{res}_{io} \cdot \left(\frac{D^{res}_{it-1}}{D^{res}_{i,-1}}\right)^{\eta^{res}_{i}} \cdot \left(\frac{p^{res}_{et}}{p^{res}_{eo}}\right)^{\eta^{res}_{ie}} \cdot \left(\frac{p^{res}_{gt}}{p^{res}_{go}}\right)^{\eta^{res}_{ig}} \cdot$$

$$\left(\frac{p^{res}_{rt}}{p^{res}_{ro}}\right)^{\eta^{res}_{ir}} \cdot (1+\pi_y)^{t\cdot\eta^{res}_{iy}} \cdot (1+\pi_h)^{t\cdot(1-\eta^{res}_{i})}$$

$$\cdot (1+\pi_{pop/h})^{t\cdot\eta^{res}_{ih}} \quad , i = g, r, e$$

Y(65) $$D^{ind}_{g/r,t} = D^{ind}_{g/r,\,o} \cdot \left(\frac{D^{ind}_{g/r,\,t-1}}{D^{ind}_{g/r,\,-1}}\right)^{\eta^{ind}_{g/r}} \cdot \left(\frac{p^{ind}_{alt,\,t}}{p^{ind}_{alt,\,o}}\right)^{\eta^{ind}_{g/r,\,alt}}$$

$$\left(\frac{p^{ind}_{et}}{p^{ind}_{eo}}\right)^{\eta^{ind}_{g/r,\,e}} \cdot \left(\frac{p^{ind}_{ct}}{p^{ind}_{co}}\right)^{\eta^{ind}_{g/r,\,c}} \cdot (1+\pi_v)^{\left(\eta^{ind}_{g/r,\,y}\right)t}$$

Y(49) $$F_{g/r,t} = \left[\frac{D^{ind}_{go}}{D^{ind}_{ro}} \cdot \left(\frac{p^{ind}_{gt}/p^{ind}_{go}}{p^{ind}_{rt}/p^{ind}_{ro}}\right)^{\eta_{g/r}}\right]^{\eta^{ind}_{g/r}} \cdot \left(F_{g/r,\,t-1}\right)^{1-\eta^{ind}_{g/r}}$$

Y(56) $$D^{ind}_{rt} = D^{ind}_{g/r,\,t} \Big/ \left(1 + F_{g/r,t}\right)$$

Y(55) $$D^{ind}_{gt} = D^{ind}_{g/r,t} - D^{ind}_{rt}$$

Y(57) $$D^{ind}_{et} = D^{ind}_{eo} \cdot \left(\frac{D^{ind}_{et-i}}{D^{ind}_{e,-1}}\right)^{\eta^{ind}_{e}} \cdot \left(\frac{p^{ind}_{et}}{p^{ind}_{eo}}\right)^{\eta^{ind}_{ee}} \cdot \left(\frac{p^{ind}_{gt}}{p^{ind}_{go}}\right)^{\eta^{ind}_{eg}} \cdot$$

$$\left(\frac{p^{ind}_{rt}}{p^{ind}_{ro}}\right)^{\eta^{ind}_{er}} \cdot \left(\frac{p^{ind}_{ct}}{p^{ind}_{co}}\right)^{\eta^{ind}_{ec}} \cdot (1+\pi_v)^{t\cdot\eta^{ind}_{ey}}$$

Y(58)-Y(60) $$D_{it}^{com} = \rho^{com} \cdot D_{it-1}^{com} + \left(\frac{p_{it}^{com}}{p_{io}^{com}}\right)^{\varepsilon_i^{com}} D_{it}^{com,*}, \quad i = g,r,e$$

Y(61) $$D_{rt}^{mv} = D_{ro}^{mv} \cdot \left(\frac{p_{rt}^{mv}}{p_{ro}^{mv}}\right)^{\eta_r^r} \cdot MV_t^{\eta_{mv}^r} \cdot GM_t^{\eta_{gm}^r} \cdot (1+\pi_{yc})^{t\cdot\eta_y^r} \cdot (1+\pi_{pop})^{(1-\eta_{mv}^r)}$$

Y(62) $$D_{rt}^{air} = D_{ro}^{air} \cdot \left[w_{pp}^{air} \cdot (1+\pi_{yc})^{t\cdot\eta_y^{air,pp}} \cdot \left(\frac{p_{rt}^{air}}{p_{rp}^{air}}\right)^{\eta_r^{air,pp}} + (1-w_{pp}^{air}) \cdot (1+\pi_{yc})^{t\cdot\eta_y^{air,f}} \right] \cdot (1+\pi_{pop})^t$$

Y(63) $$D_{rt}^{rr} = D_{ro}^{rr} \cdot \left[w_{pp}^{rr} \cdot (1+\pi_{yc})^{t\cdot\eta_y^{rr,pp}} \left(\frac{p_{rt}^{rr}}{p_{ro}^{rr}}\right)^{\eta_r^{rr,pp}} \cdot \left(\frac{p_{rt}^{air}}{p_{ro}^{air}}\right)^{\eta_{air}^{rr,pp}} + (1-w_{pp}^{rr})\ (1+\pi_{rr})^t \right] \cdot (1+\pi_{pop})^t$$

Y(64) $$D_{rt}^{w/pl} = \left[D_{ro}^{ww}\ (1+\pi_{yc})^{t\cdot\eta_y^{ww}} + D_{ro}^{pl}\ (1+\pi_{yc})^{t\cdot\eta_y^{pl}} \cdot \frac{p_{rt}^{oth}}{p_{ro}^{oth}}^{\eta_r^{pl}} \right] \cdot (1+\pi_{pop})^t$$

$$Y(48)\quad Q_{et} = \min\begin{cases}[(1+\lambda_e^{ind})\cdot D_{et}^{ind} + (1+\lambda_e^{oth})(D_{et}^{com}+D_{et}^{res})]/ \\ \qquad [1-(1+\lambda_e^{oth})\sigma_e^{oth}] \\ \bar{Q}_{et}\end{cases}$$

$$Y(66)\quad D_{et}^{oth} = \sigma_e^{oth} Q_{et}$$

$$Y(67)\quad D_{ct} = D_{co}\cdot(1+\pi_{gnp})^t \cdot \left(\frac{p_{ct}^{ind}}{p_{co}^{ind}}\right)^{\eta_c^c} \cdot \left(\frac{p_{alt,t}^{ind}}{p_{alt,o}^{ind}}\right)^{\eta_{alt}^c} \cdot e^{t\cdot\eta_t^c}$$

$$Y(74)\quad Q_{et}^c = (Q_{et}/\bar{Q}_{et})\cdot \bar{Q}_{et}^c$$

$$Y(75)\quad Q_{et}^n = (Q_{et}/\bar{Q}_{et})\cdot \bar{Q}_{et}^n$$

$$Y(76)\quad Q_{et}^r = (Q_{et}/\bar{Q}_{et})\cdot \bar{Q}_{et}^r$$

$$Y(77)\quad Q_{et}^g = (Q_{et}/\bar{Q}_{et})\cdot \bar{Q}_{et}^g$$

$$Y(78)\quad Q_{et}^{oth} = (Q_{et}/\bar{Q}_{et})\cdot \bar{Q}_{et}^{oth}$$

$$Y(68)\quad D_{gt}^{oth} = \sigma_g^{oth}\cdot(D_{gt}^{res}+D_{gt}^{com}+D_{gt}^{ind}+\alpha_g^{eu}\cdot Q_{et}^g)$$

$$Y(69)\quad D_{rt}^{oth} = \sigma_r^{oth}\cdot(D_{rt}^{res}+D_{rt}^{com}+D_{rt}^{ind}+D_{rt}^{mv}+D_{rt}^{air} + D_{rt}^{rr}+D_{rt}^{w/pl}+\alpha_r^{eu}\cdot Q_{et}^r)$$

$$Y(71)\quad G_{et} = (1+\lambda_e^{ind})D_{et}^{ind} + (1+\lambda_e^{oth})(D_{et}^{res}+D_{et}^{com}+D_{et}^{oth}) - Q_{et}$$

$$Y(72)\quad G_{gt} = D_{gt}^{res}+D_{gt}^{com}+D_{gt}^{ind}+D_{gt}^{oth}+\alpha_g^{eu}\cdot Q_e^{gt} - Q_{gt}(1+\mu_g+\mu_{lng}-\alpha_g^p) - \alpha_g^p\cdot Q_{gt}^c$$

Note: If $G_{gt} < 0$ and $c^{us}_{gt} = k^{us}_{gt}$, then all Q^{i}_{gt}, M_{gt}, and $M_{lng \cdot t}$ are reduced equiproportionally until $G_{gt} = 0$.

Y(46) $$M_{pt} = \left[D^{res}_{rt} + D^{com}_{rt} + D^{ind}_{rt} + D^{mv}_{rt} + D^{air}_{rt} + D^{rr}_{rt} + D^{w/pl}_{rt} + D^{oth}_{rt} + a^{eu}_{r} \cdot Q^{r}_{et} - Q_{pt} - (Q_{gt} - Q^{c}_{gt}) \cdot a^{p}_{g}\right] / 1 + (L_r/L_p)$$

Y(47) $$M_{rt} = (L_r/L_p) \cdot M_{pt}$$

Y(73) $$G_{pt} = 0$$

Note: In cases where petroleum imports are not determined as a residual, G_{pt} will not be identically zero. When $G_{pt} < 0$ and $c^{us}_{pt} = k^{us}_{pt}$, then all Q^{i}_{pt}, M_{pt}, and M_{rt} are reduced equiproportionally until $G_{pt} = 0$.

Y(70) $$G_{ct} = D_{ct} + a^{eu}_{e} Q^{c}_{et} + a^{sg}_{c} Q^{c}_{gt} + a^{sp}_{c} Q^{c}_{pt} - Q^{w}_{ct} - Q^{e}_{ct}$$

Note: If $G_{ct} < 0$ and $c^{e}_{ct} = k^{e}_{ct}$, then Q^{e}_{ct} and Q^{w}_{ct} are reduced equiproportionally until $G_{ct} = 0$.

T1 $$I_1 = G_{ct}/(D_{ct} - G_{ct}/2) \text{ or } (Q^{e}_{ct} + Q^{w}_{ct} + G_{ct}/2)$$

T2 $$I_2 = G_{gt}/[Q_{gt}(1 + L_g + L_{lng}) + G_{gt}/2]$$

T3 $$I_3 = G_{pt}/[Q_{pt}(1 + L_p + L_r) + G_{pt}/2]$$

T4 $$I_4 = G_{et}/(Q_{et} + G_{et}/2)$$

Note: The "step" variables, I_1-I_4, may be varied in size by an arbitrary multiplicative constant.

The Postiterative Section

Z(146) $Q_{ut} = \alpha_u Q^n_{et} + \beta_u (Q^n_{et} - Q^n_{et-1})$

Appendix B

PARAMETER VALUES

I. BASE CASE

A. Coal Supply

$$\alpha_c^{eu} = \begin{bmatrix} 10.317 \\ 10.000 \\ 9.790 \end{bmatrix} 10^3 \text{Btu/kWh}, \; t = \begin{bmatrix} 1\text{-}3 \\ 4\text{-}13 \\ 14\text{-}23 \end{bmatrix} \begin{matrix} \text{thermal heat} \\ \text{rate of coal-} \\ \text{fired power plants} \end{matrix}$$

$$\left.\begin{aligned} \alpha_c^{sp} &= 1.3 \\ \alpha_c^{sg} &= 1.5 \end{aligned}\right\} \text{ratios of input to output energy of plants for syncrude and syngas production}$$

$$\sigma_c^e = \sigma_c^w = 0.95 \quad \text{SRMC function elasticity parameter}$$

$$\left.\begin{aligned} \varepsilon_c^e &= 0.01 \\ \varepsilon_c^w &= 0.02 \end{aligned}\right\} \text{SMRC function elasticity parameter}$$

$$\tau_c^{ind} = 3.75 \text{ \$/ton} \quad \text{LRMC of transporting and distributing coal to industrial customers}$$

$$\pi_c^e = \begin{bmatrix} 0.005, \; t = \begin{bmatrix} 1\text{-}2 \\ 3\text{-}7 \\ 8\text{-}12 \\ 13\text{-}23 \end{bmatrix} \end{bmatrix} \begin{matrix} \text{maximum feasible growth} \\ \text{rate on LRMC curve for} \\ \text{eastern coal} \end{matrix}$$

$$\pi_c^w = \begin{bmatrix} 0.100, \; t = \begin{bmatrix} 1\text{-}2 \\ 3\text{-}7 \\ 8\text{-}12 \\ 13\text{-}23 \end{bmatrix} \end{bmatrix} \begin{matrix} \text{maximum feasible growth} \\ \text{rate on LRMC curve for} \\ \text{western coal} \end{matrix}$$

$\bar{Q}^{e}_{co} = Q^{e}_{co} = 13.47 \ \ 10^{15}$ Btu
$\bar{Q}^{w}_{co} = Q^{w}_{co} = 1.00 \ \ 10^{15}$ Btu
} initial coal production and capacity

$\rho^{e}_{c} = \rho^{w}_{c} = 0.5$ fraction of production in excess of *ex-ante* capacity that represents accelerated additions to capacity

$k^{e}_{co} = 8.00$ \$/ton initial LRMC, eastern coal

$k^{w}_{co} = 3.00$ \$/ton initial LRMC, western coal

$p^{ind}_{co} = 11.40$ \$/ton initial average price of industrial coal delivered to plant

$p^{ind}_{alt,o} = 0.547$ \$/10^{6} Btu initial average price of gas and oil to industrial customers

$\delta_{c} = 0.04$ depreciation rate of coal-producing capacity

$\tau^{we}_{c} = 5.00$ \$/ton LRMC of transporting western coal to east for shipment to industrial customers

$\beta^{e}_{co} = 5.00$ \$/ton
$\beta^{w}_{co} = 10.00$ \$/ton
} increase in LRMC as $Q^{e\ max}_{c}$ and $Q^{w\ max}_{c}$ are approached

$\eta^{e}_{c} = 1.0$
$\eta^{w}_{c} = 3.0$
} LRMC increase-delay parameter (higher no. implies greater delay)

$Q^{e\ max}_{c} = 2000 \ \ 10^{15}$ Btu
$Q^{w\ max}_{c} = 1000 \ \ 10^{15}$ Btu
} maximum cumulative production level consistent with LRMC of no more than $k_{o} + \beta_{o}$

$W^{w}_{co} = 0.07$ initial share of production due to western coal

B. Electricity Supply

$\lambda_e^{ind} = 0.066$
$\lambda_e^{oth} = 0.132$
} ratio of electricity generated to electricity delivered minus one

$\alpha_u = 0.026$ 10^6 tons $U_3O_8/10^{12}$ kWh
$\beta_u = 0.091$ "
} uranium usage factors for nuclear reactors

$\sigma_{eu}^c = \begin{cases} 0.4637 \text{ mills/kWh} \\ 0.4417 \text{ \$/ton Coal} \\ 0.4306 \end{cases}$, $t = \begin{cases} 1\text{-}3 \\ 4\text{-}13 \\ 14\text{-}23 \end{cases}$ fuel cost factor for coal-fired plants (incorporates thermal heat rate)

$\sigma_1^n = 0.052$
$\sigma_2^n = 0.027273$
} mills/kWh per \$/lb uranium — fuel cost factors for nuclear power plants

$k_{uo} = 10.00$ \$/lb — initial LRMC for U_3O_8 (uranium as yellow cake)

$p_{eo}^{res} = 22.9$
$p_{eo}^{com} = 22.2$
$p_{co}^{ind} = 10.9$
} mills/kWh — initial average electricity prices

$\pi_e = \begin{cases} 0.07, & t = 1\text{-}8 \\ 0.06, & 9\text{-}17 \\ 0.05, & 18\text{-}23 \end{cases}$ maximum feasible growth rate for electricity

$\underline{k}_e^c = 6.0$
$\bar{k}_e^c = 10.5$
} mills/kWh — minimum and maximum LRMCs (ex-fuel) for coal-fired generating plants

$\underline{k}_e^n = 8.0$
$\bar{k}_e^n = 14.9$
} mills/kWh — minimum and maximum LRMC's (ex-fuel) for nuclear generating plants

$Q_{eo}^c = 0.7706$
$Q_{eo}^n = 0.0540$
$Q_{eo}^r = 0.2725$
$Q_{eo}^g = 0.3757$
$Q_{eo}^{oth} = 0.2745$
} 10^{12} kWh — initial generation levels by type of energy source

$\gamma_e = 0.965$ — decay factor applied in cost averaging so as to phase out historical costs in ratemaking

$\beta_u^k = 10 \frac{\$/\text{lb } U_3O_8}{10^6 \text{ ton } U_3O_8}$ — slope of LRMC curve for U_3O_8 for values of LRMC at or above 10 \$/lb.

$\phi_e = 0.08$ — LRMC growth factor for electric generating capacity

$\beta_e^r = \begin{cases} .0261, & t = 1 \\ .0100 & t = 2\text{-}23 \end{cases}$
$\beta_e^g = -.005$
$\beta_e^{oth} = .002$
} 10^{12} kWh/yr — projected annual change in generation by oil, gas, and "other" sources

$\tau_e^{res} = 12.0 + t/3$
$\tau_e^{com} = 11.5 + t/3$
$\tau_e^{ind} = 3.0 + 3.5\ t/24$
} mills/kWh — LRMC's of electricity transmission and distribution and of customer service

$c_{nf,o} = 1.62$
$c_{nf,\,max} = 2.48$
} mills/kWh — initial and maximum values for nuclear fuel cycle costs, ex-uranium

$\bar{Q}_{eo}$ = 1.7473 10^{12} kWh — initial annual electric production capacity

Q_{uo} = 0 10^{6} tons U_3O_8 — nominal initial production of U_3O_8

γ_e^n = 0.7893
$\sigma_e^{n,\,max}$ = 0.6987
β_e^n = 21.61
} parameters in the nuclear share function

δ_e = 0.033 — annual decay factor on electricity production capacity

C. Natural Gas and Syngas

α_g^{eu} = 10.876 10^3 Btu/kWh

μ_{lng} = 0.005

μ_g = max $\begin{vmatrix} 0.040-0.005t \\ 0 \end{vmatrix}$

$\varepsilon_g^{ala} = \varepsilon_g^{us}$ = 0.0125
ε_g^{c} = 0.004
$\sigma_g^{ala} = \sigma_g^{us} = \sigma_g^{c}$ = 0.95
} SRMC function elasticity parameters

β_g^c = 0.088 tons coal/10^6 Btu — factor for computing coal gasification cost

τ_c^g = 0.0 \$/ton — cost of transporting coal from mine to gasification plant (zero at mine mouth plants)

$Q_{go}^{ala,\ max} = \hat{Q}_{go}^{ala} = \bar{Q}_{go}^{ala} = Q_{go}^{ala} = 0.1387$

$Q_{go}^{us,\ max} = \hat{Q}_{go}^{us} = \bar{Q}_{go}^{us} = Q_{go}^{us} = 24.7396$

$\bar{Q}_{go}^{c} = 0.6000$

$M_{go} = 0.9707$

$M_{lng,\ o} = 0.0000$

10^{15} Btu — initial values for annual production, annual production capacity, and imports of gas

$\rho_g^{ala} = \rho_g^{us} = 0.33$ — fraction of production in excess of *ex-ante* productive capacity, that represents accelerated additions to capacity

$t_g^c = 3$ — first feasible year for coal gasification

$p_{go}^{res} = 1.1858$

$p_{go}^{com} = 0.9058$

$p_{go}^{ind} = 0.4495$

$\$/10^6$ Btu — initial average natural gas prices

$\lambda_g^{ind} = 0.061$

$\lambda_g^{com} = 0.072$

$\lambda_g^{res} = 0.072$

ratio of gas production to gas delivered minus one

$\tau_g^{ind} = 0.2561$

$\tau_g^{com} = 0.7144$

$\tau_g^{res} = 0.9924$

$\$/10^6$ Btu — LRMC of transmission, distribution, and customer service

$R_{go}^{ala} = R_{go}^{us} = 0.0$ initial value of cumulative additions to reserves

$R_g^{ala*} = 369.5$
$R_g^{us*} = 1545.4$
10^{15} Btu — maximum cumulative additions to reserves (nominal)

$$\pi_g^{ala} = \begin{cases} 0.15, & Q_{gt-1}^{ala} < 0.21 \\ 0.50, & 0.21 \leq Q_{gt-1}^{ala} < 1.65 \\ 0.22, & 1.65 \leq \text{"} < 4.54 \\ 0.07, & 4.54 \leq \text{"} \text{ and } R_{gt-1}^{ala}/R_g^{ala*} < 1/3 \\ 0.14, & \text{"} \leq \text{"} \text{ "} \text{ "} \geq 1/3 \end{cases}$$

$$\gamma_g^{ala} = \begin{cases} 0.0, & R_{gt-1}^{ala}/R_g^{ala*} < 1/3 \\ 0.21, & \text{"} \geq 1/3 \end{cases}$$

$\phi_g = 2/3$

factors determining the maximum feasible rate of expansion along the LRMC curve

$$\pi_g^{us} = \begin{cases} 0.015, & R_{gt-1}^{us}/R_g^{us*} < 1/2 \\ 0.060, & \text{"} \geq 1/2 \end{cases}$$

$$\gamma_g^{us} = \begin{cases} 0.00, & R_{gt-1}^{us}/R_g^{us*} < 1/2 \\ 0.09, & \text{"} \geq 1/2 \end{cases}$$

determinants of maximum feasible growth rate on LRMC

$\gamma_g = 0.967$ decay factor applied in cost averaging so as to phase out historical costs in ratemaking

$Q_{g,-1}^{ala} = Q_{g,-1}^{ala} = 0.1343$
$Q_{g,-1}^{us} = Q_{g,-1}^{us} = 24.6707$
10^{15} Btu — annual production and capacity in base year minus one

$\nu_g = 8.834$ marginal reserve-production ratio

$\delta_g = 0.105$ decline rate on existing wells

$k_{go}^{ala} = 0.80$
$k_{go}^{us} = 0.40$
$\$/10^6$ Btu — LRMC of natural gas production

$\bar{k}_{gt}^{c} = 1.00$ (t = 1-13), 0.75 (t = 14-23) $\$/10^6$ Btu — LRMC of syngas production ex-coal

$\bar{c}_{lng,\,t}^{m} = 2.50$ $\$/10^6$ Btu — price of LNG imports, f.o.b., regasification plant

$\beta_g^{ala} = 0.0$
$\beta_g^{us} = 0.0007$
$\frac{\$/10^6 \text{ Btu}}{10^{15} \text{ Btu}}$ — shape of LRMC curve

$\delta_{syn} = 0.95$ — one minus the decay rate of synfuel capacity when price is below LRMC

D. Petroleum and Syncrude

$\bar{r}_r^{eu} = 11.034$ 10^3 Btu/kWh — thermal heat rate for oil-fired electric power plants

$\mu_p = 0.3510$
$\mu_r = 0.3170$
ratios of net imports to domestic production production (used only in ratio μ_r/μ_p in base case)

$\sigma_p^{ala} = \sigma_p^{us} = \sigma_p^{s} = \sigma_p^{e} = 0.95$
$\epsilon_p^{ala} = \epsilon_p^{us} = 0.01$
$\epsilon_p^{s} = \epsilon_p^{c} = 0.004$
elasticity parameters in SRMC functions

$\beta_p^{c} = 0.0765$ tons coal/10^6 Btu — factor for computing synfuel production cost

$\tau_c^p = 0.0$ \$/ton coal — cost of transporting coal from mine to syncrude plant (zero at minemouth plants)

$\beta_r^i = 1.0$ i = mv, ind, com, res, air, rr, oth — multiplicative pricing coefficient for refined petroleum products (not presently effective)

$\tau_r^{mv} = 1.96$
$\tau_r^{ind} = -0.35$
$\tau_r^{com} = 0.39$
$\tau_r^{res} = 0.39$
$\tau_r^{air} = 0.22$
$\tau_r^{rr} = 0.09$
$\tau_r^{oth} = 0.09$
} \$/10^6 Btu — transportation and distribution costs for refined petroleum products, including taxes

$Q_{po}^{ala} = Q_{po}^{ala,\,max} = \bar{Q}_{po}^{ala} = \hat{Q}_{po}^{ala} = 0.41$
$Q_{po}^{us} = Q_{po}^{us,\,max} = \bar{Q}_{po}^{us} = \hat{Q}_{po}^{us} = 18.93$
} 10^{15} Btu — annual production and capacity in base year

$\rho_p^{ala} = \rho_p^{us} = 0.33$ — fraction of production in excess of [illegible] productive capacity that represents accelerated additions to capacity

$\delta_p = 0.08$ — decline rate on existing wells

$\nu_p = 11.3625$ — marginal reserve-production ratio

$$\left.\begin{aligned} P_{ro}^{mv} &= 2.99 \\ P_{ro}^{ind} &= 0.68 \\ P_{ro}^{com} &= 1.42 \\ P_{ro}^{res} &= 1.42 \\ P_{ro}^{air} &= 1.25 \\ P_{ro}^{rr} &= 1.12 \\ P_{ro}^{oth} &= 1.12 \end{aligned}\right\} \$/10^6 \text{ Btu}$$

base year average prices for refined petroleum products

$R_{po}^{ala} = R_{po}^{us} = R_{po}^{s} = 0.0$ cumulative additions to reserves or cumulative production at start of year (nominal)

$P^{max} = 0.048$ capacity lost because of excessive pumping expressed at a fraction of production in excess of [illegible] capacity

$P^{min} = 0.0$ capacity lost because of low pumping rate

$R_p^{us*} = 905$

$R_p^{ala*} = 389$ 10^{15} Btu maximum cumulative additions to reserves (nominal)

$$P^{us} = \begin{cases} -0.031, & t = 1 \\ 0.000 & 2\text{-}3 \\ 0.010 & 4\text{-}23 \text{ and } R_{pt-1}^{us} / R_p^{us*} \leq 1/2 \\ 0.040 & " \quad " \quad " \quad > 1/2 \\ 0.040 & \end{cases}$$

$$P^{ala} = \begin{cases} 0.085, & t = 1\text{-}3 \\ 0.363 & 4\text{-}8 \\ 0.046 & 9\text{-}23 \text{ and } R_{pt-1}^{ala} / R_p^{ala*} \leq 1/2 \\ 0.184 & 9\text{-}23 \quad " \quad " \quad > 1/2 \end{cases}$$

$$\gamma_p^{us} = \begin{cases} 0.0, & R_{pt-1}^{us} \Big/ R_p^{us*} \leq 1/2 \\ 1.5 \cdot \tau_p^{us}, & R_{pt-1}^{us} \Big/ R_p^{us*} > 1/2 \end{cases}$$

$$\gamma_p^{ala} = \begin{cases} 0.0, & R_{pt-1}^{ala} \Big/ R_p^{ala*} \leq 1/2 \\ 1.5 \cdot \tau_p^{ala}, & \text{"} > 1/2 \end{cases}$$

$$\phi_p = 2/3$$

The parameters τ^{us}, τ^{ala}, γ^{us}, γ^{ala}, and ϕ_p control the determination of the maximum feasible growth rate along the LRMC curves for lower-48 and Alaskan crude

$\bar{Q}^c_{po} = 0.2$
$\bar{Q}^s_{po} = 0.4$
10^{15} Btu — initial production capacity for syncrude and shale oil production

$t_c = 8$
$t_s = 8$
initial date of production feasibility for syncrude and shale oil

$$\tau_p^c = \begin{cases} 0.476, & t = \begin{cases} t_c - 13 \\ 14-23 \end{cases} \\ 0.040 \end{cases}$$

$$\tau_p^s = \begin{cases} 0.149, & t = \begin{cases} t_s - 13 \\ 14-23 \end{cases} \\ 0.040 \end{cases}$$

maximum feasible expansion rates on LRMC curve

$\beta_p^{ala} = 0.0$
$\beta_p^{us} = 0.001428$
$\beta_p^{s} = 0.000458$
$\frac{\$/10^6 \text{ Btu}}{10^{15} \text{ Btu}}$ — slope of LRMC curve

$\bar{k}^c_{pt} = 0.85$ $\$/10^6$ Btu — LRMC of syncrude ex-coal cost

$\bar{c}_{rt} = 0.50$ $\$/10^6$ Btu — LRMC of refining

$k_{po}^{ala} = 0.54$
$k_{po}^{us} = 0.67$
$k_{po}^{s} = 1.07$
} $/10^6 Btu — initial LRMC of crude, f.o.b. p.o.e., wellhead, or shale plant

$\alpha_g^p = 0.1037$ — fraction of lower-48 and Alaskan gas production transferred to petroleum sector as natural gas liquids

E. Residential Demand

$D_{go}^{res} = 5.28$
$D_{g-1}^{res} = 5.13$
$D_{ro}^{res} = 2.94$
$D_{r-i}^{res} = 2.84$
} 10^{15} Btu

$D_{eo}^{res} = 0.5114$
$D_{e-1}^{res} = 0.4791$
} 10^{12} kWh

} initial-year and initial-year-minus-one values for energy consumption

$\eta_g^{res} = 0.8859$ $\eta_e^{res} = 0.8859$ $\eta_r^{res} = 0.9000$ — lag coefficient

$\eta_{ge}^{res} = 0.0319$ $\eta_{ee}^{res} = -0.1278$ $\eta_{re}^{res} = 0.0210$
$\eta_{gg}^{res} = -0.2362$ $\eta_{eg}^{res} = 0.0924$ $\eta_{rg}^{res} = 0.2100$
$\eta_{gr}^{res} = 0.0491$ $\eta_{er}^{res} = 0.0114$ $\eta_{rr}^{res} = -0.1956$
$\eta_{gy}^{res} = 0.1620$ $\eta_{ey}^{res} = 0.0913$ $\eta_{ry}^{res} = 0.0379$
$\eta_{gh}^{res} = 0.1415$ $\eta_{eh}^{res} = -0.1073$ $\eta_{rh}^{res} = 0.0000$

short-run elasticity coefficients for prices, income, and household size

$\pi_y = 0.0224$
$\pi_h = -0.0065$

annual growth rates for real personal income per household and household size

F. Commercial Demand

$D_{go}^{com} = 0.55$ 10^{15} Btu
$D_{ro}^{com} = 0.66$ 10^{15} Btu
$D_{eo}^{com} = 0.405$ 10^{12} kWh

initial-year values for energy consumption

$\rho^{com} = 0.97$ fraction of last year's floor space that is neither replaced nor refitted

$\varepsilon_g^{com} = \varepsilon_r^{com} = \varepsilon_e^{com} = -0.30$ long-run elasticity of energy intensity with respect to price

α_j^i's

j \ i	1	2	3	4	5	units
g	+0.49	-0.06	-0.10	-0.24	-0.10	$/$10^6$ Btu
r	+0.07	0.00	0.00	0.00	+0.20	$/$10^6$ Btu
e	+6.5	+1.7	-2.5	-2.8	-4.0	mills/kWh

regional price differences (deviations from national average)

β_{j1}^i's (all electric)

j \ i	1	2	3	4	5	units
f	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
e	.0250	.0267	.0246	.0296	.0217	10^3 kWh/ft^2

β_{j2}^i's (conventional)

j \ i	1	2	3	4	5	units
f	.046	.053	.040	.073	.027	10^6 Btu/ft^2
e	.0155	.0158	.0164	.0147	.0161	10^3 kWh/ft^2

energy-use coefficients (at base-year prices)

β_j^i3's (total energy)

j \ i	1	2	3	4	5	units	
f	.181	.191	.184	.197	.171	10^6 Btu/ft^2	energy use coefficients (at base-year prices)
e	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	

$A_o^1 = 6.144$
$A_o^2 = 6.440$
$A_o^3 = 6.053$
$A_o^4 = 0.842$
$A_o^5 = 3.277$

10^9 ft^2 — inventory of commercial floor space in base year minus two (1970)

$\gamma_a^1 = 0.045$
$\gamma_a^2 = 0.046$
$\gamma_a^3 = 0.052$
$\gamma_a^4 = 0.050$
$\gamma_a^5 = 0.047$

projected annual rate of growth of commercial floor space

$\bar{S}_j^i t = 0.8$ for all i, j, t — maximum saturation of system type j in region i

$\underline{S}_j^i t = 0.1$ for all i, j, t — minimum saturation of system type j in region i

$\gamma^i = 3.17$, i = 1, ..., 5 — gas-oil substitution coefficient

$m_1 = -3.1704$ $\$/10^6$ Btu

$n_1 = 0.2345$ $(\$/10^6 \text{ Btu})/(\text{mills/kWh})$

$m_2 = -2.5061$ $\$/10^6$ Btu

$n_2 = 0.1028$ $(\$/10^6 \text{ Btu})/(\text{mills/kWh})$

} parameters of the normalized equal-cost curves

G. Industrial Demand

$\eta^{ind}_{g/r} = 0.900$ $\eta^{ind}_{e} = 0.900$ lag coefficients

$\eta^{ind}_{g/r,\,e} = 0.030$ $\eta^{ind}_{ee} = -0.102$

$\eta^{ind}_{g/r,\,c} = 0.030$ $\eta^{ind}_{ec} = 0.018$

$\eta^{ind}_{g/r,\,alt} = -0.100$ $\eta^{ind}_{eg} = 0.015$

$\eta^{ind}_{g/r,\,y} = 0.033$ $\eta^{ind}_{er} = 0.012$

$\eta^{ind}_{ey} = 0.064$

} short-run elasticities of demand with respect to prices and output (real value added)

$D^{ind}_{g/r,\,o} = 15.92$

$D^{ind}_{g/r,\,-1} = 15.01$

$D^{ind}_{go} = 10.25$

$D^{ind}_{ro} = 5.67$

} 10^{15} Btu — initial-year and initial-year-minus-one values for energy consumption

$D^{ind}_{eo} = 0.6395$

$D^{ind}_{e-1} = 0.5927$

} 10^{12} kWh

$\eta_{g/r} = -3.0$ gas-oil substitution coefficient

$\pi_v = 0.035$ projected annual growth rate for real value added in the industrial sector

H. Transportation Demand

$D_{ro}^{mv} = 12.345\ 10^{15}$ Btu — initial-year motor vehicle energy consumption

$MV_o = 1$ — ratio of motor vehicle stock in year zero to initial-year motor vehicle stock

$VH_o = 1.453$ — vehicles per household in base year

$\eta_r^r = -0.104$
$\eta_y^r = 0.189$
$\eta_{mv}^r = 0.805$
$\eta_{gm}^r = -0.650$
— elasticity coefficients in gasoline demand equation

$\pi_{yc} = 0.0291$
$\pi_{pop} = 0.0094$
— projected annual rates of growth for real personal income per capita and population

$\eta_r^{gm} = 0.17015$ — elasticity coefficient in gasoline mileage equation

$\eta_o^{mv} = 0.18752$
$\eta_y^{mv} = 0.29958$
$\eta_r^{mv} = -0.06791$
$\eta^{mv} = 0.76924$
— coefficients in motor vehicle stock projection equation

$D_{ro}^{air} = 1.428\ 10^{15}$ Btu — base-year energy consumption due to commercial air transportation

$w_p^{air} = 0.744$ — share of commercial air energy consumption due to passenger traffic

$\eta_y^{air,\,p} = 1.97$
$\eta_r^{air,\,p} = -0.11$
$\eta_y^{air,\,f} = 3.24$
$\eta_r^{air,\,f} = -0.56$

elasticity coefficients in air transportation energy demand equations

$D_{ro}^{rr} = 0.521 \quad 10^{15}$ Btu — base-year energy use due to rail transportation

$w_p^{rr} = 0.038$ — share of railroad energy consumption due to passenger traffic

$\eta_y^{rr,\,p} = 1.44$
$\eta_{air}^{rr,\,p} = 0.02$
$\eta_r^{rr,\,p} = -0.10$

elasticity coefficients in rail passenger transportation energy demand equations

$\pi_{rr} = 0.023$ — projected annual rate of growth of rail freight transportation per capita

$D_{ro}^{w} = 0.705$
$D_{ro}^{pl} = 0.845$

10^{15} Btu — base-year energy use by waterways and pipelines

$\eta_y^{w} = 0.51$
$\eta_y^{pl} = 0.24$
$\eta_r^{pl} = -0.21$

elasticity coefficients for waterway/oil pipeline energy use equation

I. Miscellaneous Demands

$\sigma_e^{oth} = 0.014$
$\sigma_g^{oth} = 0.153$
$\sigma_r^{oth} = 0.125$

ratio of miscellaneous demands to all other demands

$D_{eo} = 6.52 \quad 10^{15}$ Btu — base-year coal consumption by all sectors except electric utilities

$\eta_t^c = -0.0348$
$\eta_c^c = -0.614$
$\eta_{alt}^c = 0.30$

coefficients in coal demand equation

$\eta_{gnp} = 0.0388$ — projected growth rate of real GNP

J. Other

$\phi_1^{w/e} = 0.5$

$\phi_2^{w/e} = 2.0$

$$\phi_c^w = \begin{cases} 1 & \text{if } Q_{ct-1}^w \; Q_{ct-1}^e \cdot \phi_1^{w/e} \\ \dfrac{1}{1 + \left| \left(Q_{ct-1}^w / Q_{ct-1}^e \right) - \phi_2^{w/e} \right| / \phi^{w/e}} & \text{otherwise} \end{cases}$$

where ϕ_c^w is the fraction of western coal consumed in the west.

II. Pessimistic Case

A. Coal Supply

$$\varepsilon_c^e = 0.005$$

$$\varepsilon_c^w = 0.005$$

$$\Pi_c^e = \begin{bmatrix} 0.005 \\ 0.010 \\ 0.020 \\ 0.040 \end{bmatrix}, \quad t = \begin{bmatrix} 1\text{-}2 \\ 3 \\ 4 \\ 5\text{-}23 \end{bmatrix}$$

$$\Pi_c^w = \begin{bmatrix} 0.100 \\ 0.075 \\ 0.050 \end{bmatrix} \quad t = \begin{bmatrix} 1\text{-}7 \\ 8\text{-}17 \\ 18\text{-}23 \end{bmatrix}$$

$$k_{co}^e = 10.00 \quad \$/\text{ton}$$

$$k_{co}^w = 4.00 \quad \$/\text{ton}$$

$$\eta_c^w = 1.0$$

B. Electricity Supply

$$\Pi_e = \begin{bmatrix} 0.067 \\ 0.060 \\ 0.050 \end{bmatrix} \quad t = \begin{bmatrix} 1 \\ 2\text{-}8 \\ 9\text{-}23 \end{bmatrix}$$

$$\left.\begin{aligned} k_e^c &= 9.0 \\ \bar{k}_e^c &= 15.0 \\ k_e^n &= 12.0 \\ \bar{k}_e^n &= 20.0 \end{aligned}\right\} \quad \text{mills/kWh}$$

$$k_{uo} = 12 \quad \$/\text{ton } U_3O_8$$

$$\beta_u^k = 12 \quad (\$/\text{ton})/10^6 \text{ ton } U_3O_8$$

$$\gamma_e^n = 0.8393$$

C. Natural Gas and Syngas

$$\mu_{lng} = 0.0025 \cdot t$$

$$\epsilon_g^{ala} = 0.006$$

$$\epsilon_g^{us} = 0.006$$

$$\epsilon_g^{c} = 0.002$$

$$\Pi_g^{us} = \begin{cases} 0.015 \\ 0.010 \\ 0.040 \end{cases}, \quad t = \begin{cases} 1 \\ 2\text{-}23 \text{ and } R_{gt-1}^{us}/R_g^{us*} < 1/2 \\ \text{otherwise} \end{cases}$$

$$\gamma_g^{us} = \begin{cases} 0.0 \\ 0.06 \end{cases}, \quad \begin{cases} R_{gt-1}^{us} \; R_g^{us*} < 1/2 \\ \text{otherwise} \end{cases}$$

$$\left.\begin{aligned} R_g^{us*} &= 900 \\ R_g^{ala} &= 250 \end{aligned}\right\} \; 10^{15} \text{ Btu}$$

$$\beta_g^{us} = 0.001 \quad (\$/10^6 \text{Btu})/10^{15} \text{ Btu}$$

$$k_{go}^{us} = 0.50 \quad \$/10^6 \text{ Btu}$$

D. Petroleum and Syncrude

$$\epsilon_p^{ala} = 0.005$$

$$\epsilon_p^{us} = 0.005$$

$$\epsilon_p^{s} = 0.002$$

$$\epsilon_p^{c} = 0.002$$

$$\left.\begin{aligned} R_p^{us*} &= 756 \\ R_p^{ala*} &= 322 \end{aligned}\right\} \; 10^{15} \text{ Btu}$$

$$\left.\begin{aligned} k_{po}^{us} &= 1.25 \\ k_{po}^{ala} &= 0.72 \end{aligned}\right\} \; \$/10^6 \text{ Btu}$$

Appendix C

COMPARISON OF THE MODEL WITH OTHER ENERGY MODELS[1]

To better demonstrate the character of the model and its unique capacities for investigating national energy policies, this appendix contrasts it with three other models--the Federal Energy Administration's energy model, the Brookhaven model, and the Hudson-Jorgenson model.[2] These are three of the best-known and most respected models currently available for evaluating energy policy.

THE FEDERAL ENERGY MODEL

The energy model being used by the Federal Energy Administration (FEA) is an augmented linear programming model of the energy sector of the U.S. economy. It takes into account regional production, conversion, and use of energy, as well as the intra- and interregional transportation of energy. Through the incorporation of suitable constraints, it also recognizes limits on environmental degradation and the regional or national supply of certain productive inputs such as capital, manpower, or special materials. The key features of the model are (1) approximation of energy-supply curves by step functions representing quantities supplied at various prices, and (2) an augmented computer procedure that permits the consistent consideration of price-responsive demand functions. This latter procedure consists of an iterative procedure whereby (1) a demand price for determining demand quantities is postulated; (2) the linear program is solved and a supply price is obtained; (3) the demand price is adjusted toward the supply price; (4) the linear program is run. The augmented linear programming model thus determines equilibrium prices while allowing for both demand and supply responsiveness.

This model is an excellent vehicle for the initial efforts of the FEA--permitting the identification, for any given year, of the critical

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[1] I am grateful to James Stucker of The Rand Corporation, who assembled the material on the models described below. I am, however, responsible for any error or inaccuracies that may remain.

[2] See Refs. 14, 15, and 16.

points of the energy system. It pinpoints the binding constraints, and estimates the value of marginal investments to alleviate these shortages or bottlenecks. This information is valuable in the formulation of federal measures affecting energy supply, from production through processing and transport. Measures affecting energy demand can be investigated by exogenously altering the demand price-quantity functions for the various types of fuel. Tax policies can be investigated through the model, and other economic and environmental impacts can be analyzed by incorporating suitable constraints on the linear programming model.

The major limitations of the FEA energy model arise from the fact that it is a static model of long-run economic equilibrium. It indicates the characteristics and sensitivities of the energy sector at a given time. It cannot be used to investigate the dynamic interactions among the various elements of the energy sector, or the short-run adjustments necessary for the successful attainment of long-run equilibrium. Moreover, the model cannot indicate how the present sectoral specification has been shaped by previous industrial actions or governmental policies. Nor can it predict how present actions may affect an evolving future scenario. More important, this type of model cannot specify either the short-run or long-run effects of exogenous shocks to the energy system--such as an Arab oil embargo or the invention of a more efficient method of coal gasification. The analysis of these situations requires a dynamic model that incorporates short-run adjustments and inefficiencies. Finally, the FEA energy model is massive, requiring a large amount of manpower and computer time for its operation.

THE BROOKHAVEN ENERGY MODEL

The energy model developed at the Brookhaven National Laboratory has two main components--a "reference energy system" and a linear programming optimization model of the U.S. energy system. The reference energy system is a network representation of all technical activities required to supply various forms of energy to end-use activities. Technologies are defined for specific fuels to represent their extraction, refinement, conversion, transport, distribution, and utilization. Each activity is represented by a link in the network. An efficiency parameter,

a cost coefficient, and several environmental impact parameters are specified for each. Reference energy systems have been developed for a number of future years to assess a broad set of technologies ranging from solar energy to coal. The network is exercised by specifying a set of end-use demands. Energy flows through the system are then computed and total environmental effects, total resource consumption, and total cost are tabulated.

The reference energy systems are used to evaluate new technologies by perturbation analysis. A new technology may involve simple replacement of an individual process or it may dictate an entirely new path in the energy system network. Technical experts must determine the efficiency, cost, and environmental parameters that describe the new technology and the degree that it may be implemented in the planning year. After these parameters have been inserted into the system, total resource consumption, cost, and environmental effects are recalculated.

The linear programming optimization model of the U.S. energy system includes the same input parameters as the reference energy systems. The model shows the full feasible range of interfuel substitutability. The solutions obtained from the model indicate the optimal demand-supply configuration of the energy system within specified constraints on resources, demands, and environmental impacts. The optimization may be performed with respect to cost, any of the environmental effects, or some combination of these factors.

The Brookhaven models focus on the technical aspects of energy extraction and production, and compare the aggregate effects of alternative supply patterns. The models perform this task well. Their applicability is, however, limited when it comes to analyzing questions of energy consumption and conservation, or the broader issues of public energy policy. Energy prices are not recognized by the models; demand for the various forms of energy is given exogenously; and no trade-offs are recognized among the various forms of energy demanded. Finally, these models are also static models of long-term market equilibrium. They cannnot be used to consider problems of short-run adjustment of either supply or demand.

THE HUDSON-JORGENSON MODEL

Hudson and Jorgenson devised an econometric model of the U.S. economy with special emphasis on the energy sectors. This model synthesizes input-output analysis and macroeconometric modeling. Input-output analysis is useful for conducting detailed analyses of supply. It is limited, however, in that technology is considered to be fixed, so no trade-offs among inputs are possible, whatever the time span under consideration. Econometric models of the national economy can be useful in studying the impact of economic policies on aggregate demand, but they usually do not provide an adequate basis for assessing the impact of these policies on supply. Hudson and Jorgenson have combined these two approaches, constructing a single model containing both macroeconometric elements and an interindustry submodel. The principal innovation of their model is that it treats the input-output coefficients as endogenous variables rather than as exogenously given parameters. That is, the input-output coefficients are estimated within the model; they depend primarily on energy prices and on product and factor prices, and may therefore be expected to change from year to year.

The Hudson-Jorgenson model includes four industrial sectors producing capital and consumer goods (agriculture, manufacturing, transportation, and trade) and five sectors producing some form of energy (coal mining, crude petroleum and natural gas, petroleum refining, electric utilities, and gas utilities). All of these sectors are considered to consume energy, materials, capital, and labor, and they are all (except the crude-oil sector) considered to produce products that are consumed both by the other sectors and by households. The model also analyzes the national aggregates of capital stock, business investment, imports and exports, total production, labor and various prices. It is made dynamic by links between investment and changes in capital stock and by preferences between present and future consumption, which determine the allocation of income between saving and consumption.

The behavioral equations of the macroeconometric growth model were estimated from historical data for the United States for the period 1929 through 1971. The interindustry model was estimated from annual data on interindustry transactions, capital and labor services, and competitive imports for the period 1947 through 1971.

The Hudson-Jorgenson model should be useful in addressing certain questions of energy policy. It combines the analyses of supply obtainable from input-output analysis with the analyses of demand response and equilibrium prices obtainable from an econometric model. And it directly relates energy supply and demand to the overall performance of the national economy.

However, the major shortcomings of this model are (1) its aggregate handling of energy sources, and, again, (2) an inability to illuminate the short-run problems due to either exogenous economic shocks to the energy system or planned governmental policies. The aggregation of supply sources is severe and precludes the analysis of many energy policies currently under consideration by the federal government. "Coal mining" encompasses both eastern and western coal. "Crude petroleum and natural gas" is an industry containing both crude petroleum and natural gas produced anywhere from Texas to the North Slope of Alaska. Such aggregation obscures differences in production responses for different fuels and different locations. Also, as the relations were econometrically estimated from historical data, it is difficult to use them to investigate the implications of new sources of supply, such as shale oil or the production of gasoline substitutes from coal or organic waste products.

THE PRESENT MODEL

The model described in this report is a dynamic simulation model of the energy sector of the U.S. economy. It considers the demand and supply of all major types of energy and can be used to trace the time path of energy prices, quantities, and imports. The key features of the model are the interactions between short-run and long-run forces (1) on supply, by considerations relating prices to short-run marginal or average costs and short-run costs to capacity levels and long-run costs, and also on the feedback from present production levels to adjustments in capacity; and (2) on demand, through changes in the stocks as well as the intensity of use of energy-consuming appliances. The model is useful in the exploration of policy options because it is dynamic and because it illuminates short-run adjustments and problems. Its assets correspond strongly with the weaknesses of the other models.

Appendix D

FREIGHT-TRANSPORT DEMAND ELASTICITIES

by James P. Stucker

Freight-transport demand elasticities were estimated from 17-year time series data on ton-miles carried, revenue, and GNP. Log-linear regressions were estimated for air, railroad, water, and pipeline transportation. Table D-1 presents the estimates.

Statistically significant elasticities from Eqs. (D-5), (D-6), and (D-8) are used in the demand model. Rail demand is based on a simple geometric time-trend for the period 1961 through 1970.

To convert price (rate) elasticities into energy price elasticities, we assumed that

$$Q = f(P) \quad \text{(demand function)}, \tag{D-1}$$

$$P = g(C) \quad \text{(pricing function)}, \tag{D-2}$$

$$C = \sum_i v_i X_i \quad \text{(cost function)}, \tag{D-3}$$

for a particular mode of transport where

Q is the quantity demanded,

P is the freight rate,

C is the cost of transport,

X_i is the amount of input i required to produce one unit of output (assumed constant),

v_i is the price of input i.

Table D-1

FREIGHT TRANSPORTATION DEMAND, 1954-1970, U.S. DATA

Log-Linear Regressions
Dependent Variable is Ton-Miles Per Capita

Equation	Mode	Intercept	Gross National Product Per Capita (1967 $)		Own Rate (1967 $)		Equation F	$\bar{R}^2$	Durbin Watson
			Elasticity	t Value	Elasticity	t Value			
D-1	Air	-2.52	3.35	34.31	--	--	1177.4	0.99	1.51
D-2	Pipeline	-1.19	1.05	11.13	--	--	123.8	0.88	0.48
D-3	Rail	6.48	0.21	2.13	--	--	4.5	0.19	0.54
D-4	Water	3.01	0.51	5.35	--	--	28.7	0.63	1.28
D-5	Air	-23.36	3.24	29.76	-2.81	1.85	684.7	0.99	1.66
D-6	Pipeline	4.18	0.24	2.32	-1.03	8.43	386.7	0.98	2.06
D-7	Rail	4.44	0.44	1.01	0.50	0.55	2.3	0.17	0.60
D-8	Water	2.79	0.55	1.47	0.03	0.09	13.4	0.61	1.30

Source: All data on domestic air transport, oil pipelines, Class I and II railroads, inland waterway carriers, GNP, population, and the wholesale price index taken directly from Ref. 17.

Now, if we consider X_j as fuel and v_j as its price, we wish to estimate the elasticity of Q with respect to v_j.

Combining Eqs. (D-1), (D-2), and (D-3), we have

$$Q = f\left[g\left(\sum_i v_i X_i\right)\right] \tag{D-4}$$

so that

$$E_{Q,v_j} = \frac{\partial Q}{\partial v_j}\frac{v_j}{Q} = f'g'\frac{v_j X_j}{Q} \quad . \tag{D-5}$$

But

$$E_{Q,P} = f'\frac{P}{Q} \quad , \tag{D-6}$$

so

$$E_{Q,v_j} = E_{Q,P}\, g'\frac{v_j X_j}{P} \quad . \tag{D-7}$$

If we consider that the mode engages in direct, cost-based pricing such that $g' = 1$, the elasticity of quantity with respect to the price of fuel is simply the rate elasticity times the ratio of fuel costs to the rate. The transport fuel elasticities in the demand model are based on this simple relationship:

$$\hat{E}_{Q,f_j} = E_{Q,P}\frac{v_j X_j}{P} \tag{D-8}$$

The estimates of $v_j X_j/P$ are taken from Ref. 15 and are shown in Table D-2

Table D-2

RATIO OF FUEL COSTS TO TRANSPORT RATES

Transportation Mode	Air	Pipeline	Rail	Water
Passenger Transport	0.10	---	0.04	---
Freight Transport	0.20	0.20	0.04	0.08

Source: Ref. 18.

Appendix E

INDUSTRIAL AND OTHER DEMAND FOR COAL

This appendix describes the procedures used to estimate the demand for coal for all purposes except generation of electricity. The form of the estimated equation is as follows:

$$\left(\frac{D^c_t}{GNP_t}\right) = \left(P^{ind}_{c,t-1}\right)^{\eta_c} \cdot \left(P^{ind}_{alt,t-1}\right)^{\eta_{alt}} \cdot e^{\eta_v \cdot t} \cdot K_c \,, \qquad \text{(E-1)}$$

where D^c_t is total coal consumption plus net exports minus coal used at electric utility; GNP_t is real gross national product; $P^{ind}_{c,t-1}$ is the average price of coal to industrial users (lagged one year); $P^{ind}_{alt,t-1}$ is the average price index of gas and oil to industrial users (lagged one year); t is time measured in years; and K_c is a constant. The prices of coal and alternative fuels are deflated by the wholesale price index. The price of gas is given a weight of 0.663 in the alternative-fuels price index, based upon 1969 consumption shares in the industrial sector. The average price of coal is taken as the wholesale price of screenings for industrial use, f.o.b. the mine, plus the average freight charge per ton of coal shipped by rail.

The parameters of Eq. (E-1) were estimated by ordinary least squares using annual data for 1961 through 1973 (1960 through 1972 for prices). In addition to the normal or "unconstrained" case, the equation was estimated once where η_{alt} was set equal to 0.3 and once where η_{alt} was 0.6. I felt justified in attempting this procedure because of the obvious improbability that the cross-price elasticity of demand would be zero or negative. Table E-1 summarizes the results for the two constrained cases. The R^2's of the two equations are not comparable, owing to the redefinition of the dependent variable in each case--the alternative-fuel price term being shifted to the left side of the equation.

Although not reported here, the unconstrained results are very close to those obtained in Eq. E-1a in Table E-1, the only exception

Table E-1

REGRESSION RESULTS FOR COAL DEMAND

Equation	Price of Coal	Price of Alternative Fuels	Time	Constant	R^2 and DW Statistic
E-1a	-0.6140	0.3	-0.03475	1.691	0.89
	(-3.132)	--	(-5.211)	(4.199)	2.40
E-1b	-0.6133	0.6	-0.03374	1.696	0.88
	(-2.961)	--	(-4.789)	(3.986)	2.38

NOTE: Figures in parentheses are t-ratios. The degrees of freedom are 10.

being a negative and statistically insignificant estimate of η_{alt}. The time-trend estimates obtained in all equations may reflect technical change--or more likely--a prolonged lagged response to the increased availability of natural gas in the post-war period.

REFERENCES

1. Anderson, K. P., *Residential Energy Use: An Econometric Analysis*, The Rand Corporation, R-1297-NSF, October 1973.

2. Mount, T. D., L. D. Chapman, and T. J. Tyrrell, *Electricity Demand in the United States: An Econometric Analysis*, Oak Ridge National Laboratory, ORNL-NSF-EP-49, June 1973.

3. Erickson, E. W., R. M. Spann, and R. Ciliano, *Fossil Fuel Energy Demand Analysis*, Decision Sciences Corporation, March 26, 1973.

4. Salter, R. G., R. L. Petruschell, and K. Wolf, *Energy Conservation in Non-Residential Buildings*, The Rand Corporation, R-1623-NSF, forthcoming.

5. Baughman, M. L., P. L. Joskow, and F. S. Zehoot, *Interfuel Substitution in the Consumption of Energy in the United States*, Massachusetts Institute of Technology Energy Laboratory, MIT-EL 74-002, May 1974.

6. Wildhorn, S., B. K. Burright, J. H. Enns, and T. F. Kirkwood, *How To Save Gasoline: Public Policy Alternatives for the Automobile*, The Rand Corporation, R-1560-NSF, October 1974.

7. Lave, L. B., "The Demand for Intercity Passenger Transportation," *J. Regional Science*, Vol. 12, April 1972, pp. 71-84.

8. Anderson, K. P., and J. C. DeHaven, *The Long-Run Marginal Costs of Energy*, The Rand Corporation, R-1590-NSF, February 1975.

9. Adelman, M., *The World Petroleum Market*, Johns Hopkins University Press, Baltimore, 1972.

10. National Petroleum Council, *U.S. Energy Outlook: Summary Volume*, December 1972.

11. Adelman, M., et al., "Energy Self-Sufficiency: An Economic Evaluation," *Tech. Review*, Vol. 76, No. 6, May 1974, pp. 22-58.

12. U.S. Atomic Energy Commission, *Nuclear Fuel Supply*, Wash-1242, Washington, D.C., May 1973.

13. Public Service Commission of Wisconsin, "Application of Madison Gas and Electric Company for Authority To Increase Its Gas and Electricity Rates--Findings of Fact and Order," August 8, 1974 (mimeographed).

14. The Federal Energy Administration, *Project Independence Report*, November 1974.

15. Hoffman, K. C. (ed.), *Energy Systems Analysis and Technology Assessment Program*, U.S. Atomic Energy Commission, Office of Planning and Analysis, Annual Report Fiscal Year 1973, Energy Systems Analysis Group, Brookhaven National Laboratory, Associated Universities, Inc., June 1973.

16. Hudson, E. A., and D. W. Jorgenson, "United States Energy Policy and Economic Growth, 1975-2000," *Bell J. Economics and Management Science*, Vol. 5, No. 2, Autumn 1974, pp. 461-514.

17. U.S. Bureau of the Census, *Statistical Abstract of the United States* (various editions), Washington, D.C.

18. Mooz, W. E., *The Effect of Fuel Price Increases on Energy Intensiveness of Freight Demand*, The Rand Corporation, R-804-NSF, December 1971.